Solutions #1

(1) a) $\binom{30}{3} = 4060$

b) * There are 28 3-element sets consisting of three consecutive numbers

$\{1,2,3\}, \ldots, \{28,29,30\}$

* Subsets with two consecutive numbers

$\{1,2,x\}$ 27 choices for x

$\{2,3,x\}$ 26 choices for x

$\{3,4,x\}$ 26 choices for x

$\vdots$

$\{28,29,x\}$ 26 choices for x

$\{29,30,x\}$ 27 choices for x

$$2(27) + 27(26) = 756$$

* Subsets with no two consecutive elements

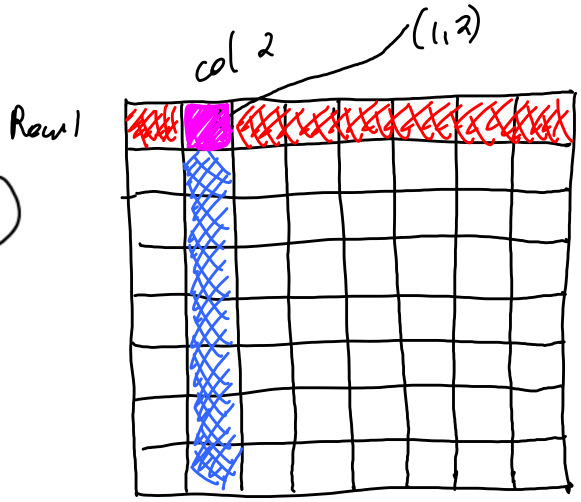
All subsets − subsets w/ 2 consecutive elements − subsets w/ 3 consecutive elements =

$$4060 - 756 - 28 = \boxed{3276}$$

c) $\binom{15}{2}\binom{15}{2} = 11{,}025$

(2)

(b)



Rook in (1,2) square OR no rook in (1,2) square

$$\binom{7}{4}P(7,4) + \binom{7}{1}\binom{7}{1}\binom{6}{3}P(6,3) =$$

$$29,400 + 117,600 =$$

$$147,000$$

(a) $\binom{8}{5}P(8,5) = \boxed{367,320}$

(c) $367,320 - 147,000 = \boxed{229,320}$

(3) a) $\binom{54}{5} = 3,162,510$

b) No Jokers OR 1 Joker OR 2 Jokers

5,5,5,A,A   5,5,A,A,J   impossible

$$\binom{13}{2}\binom{2}{1}\binom{4}{3}\binom{4}{2} + \binom{2}{1}\binom{13}{2}\binom{4}{2}\binom{4}{2} + 0$$

$$3744 + 5616 + 0$$

$$9360$$

c) No Jokers $\quad 10 \cdot 4 = 40$

OR

One Joker in sequences A-5, ..., 9-K. Joker cannot be the low card. $\quad 2 \cdot 9 \cdot 4 \cdot 4 = 288$

One Joker in sequence 10-A. The Joker may be any place in the sequence $\quad 2 \cdot 5 \cdot 4 = 40$

Two Jokers in sequences A-5, ..., 9-K, Joker cannot be the low card $\quad 9 \cdot \binom{4}{2} \cdot 4 = 216$

Two Jokers in sequence 10-A. Either Joker may be any place in the sequence $\quad \binom{5}{2} \cdot 4 = 40$

Total

624

(4) $M = \{3\cdot a, 4\cdot b, 5\cdot c\}$ has 12 elements.

<u>11-permutations</u>

Each 11-permutation yields a 12-permutation by placing the missing element last.

Conversely, each 12-permutation yields an 11-permutation by removing the last element.

$$\binom{12}{3,4,5} = 27{,}720$$

<u>10-permutations</u>

The 10-element subsets of M are

$\{1\cdot a, 4\cdot b, 5\cdot c\}$ $\{3\cdot a, 2\cdot b, 5\cdot c\}$ $\{3\cdot a, 4\cdot b, 3\cdot c\}$

$\{2\cdot a, 3 b, 5\cdot c\}$ $\{2\cdot a, 4\cdot b, 4\cdot c\}$ $\{3\cdot a, 3\cdot b, 4\cdot c\}$

$$\binom{10}{1,4,5} + \binom{10}{3,2,5} + \binom{10}{3,4,3} + \binom{10}{2,3,5} + \binom{10}{2,4,4} + \binom{10}{3,3,4} = 17{,}850$$

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(5) Distribute 10 identical balls into 3 distinct baskets.

$$\binom{10+3-1}{3-1} = \binom{12}{2} = 66$$