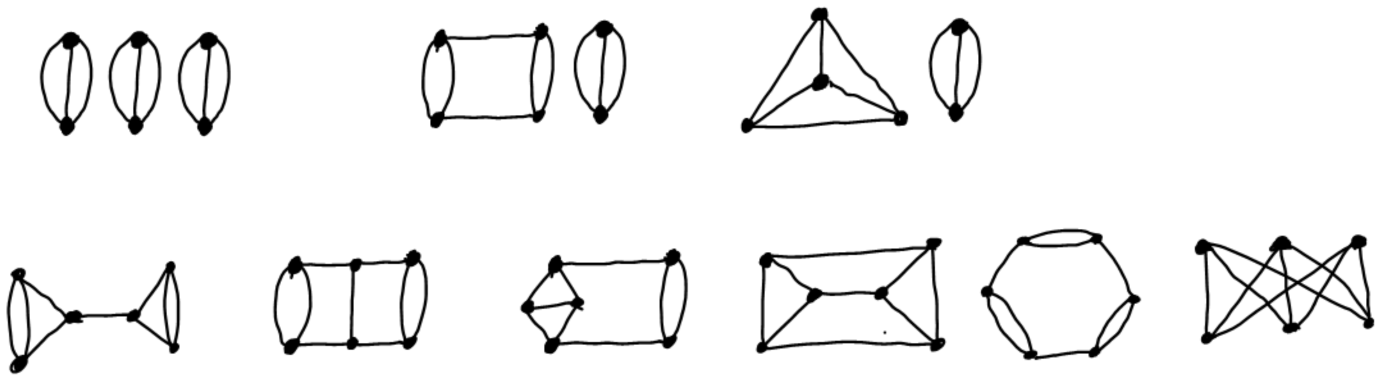
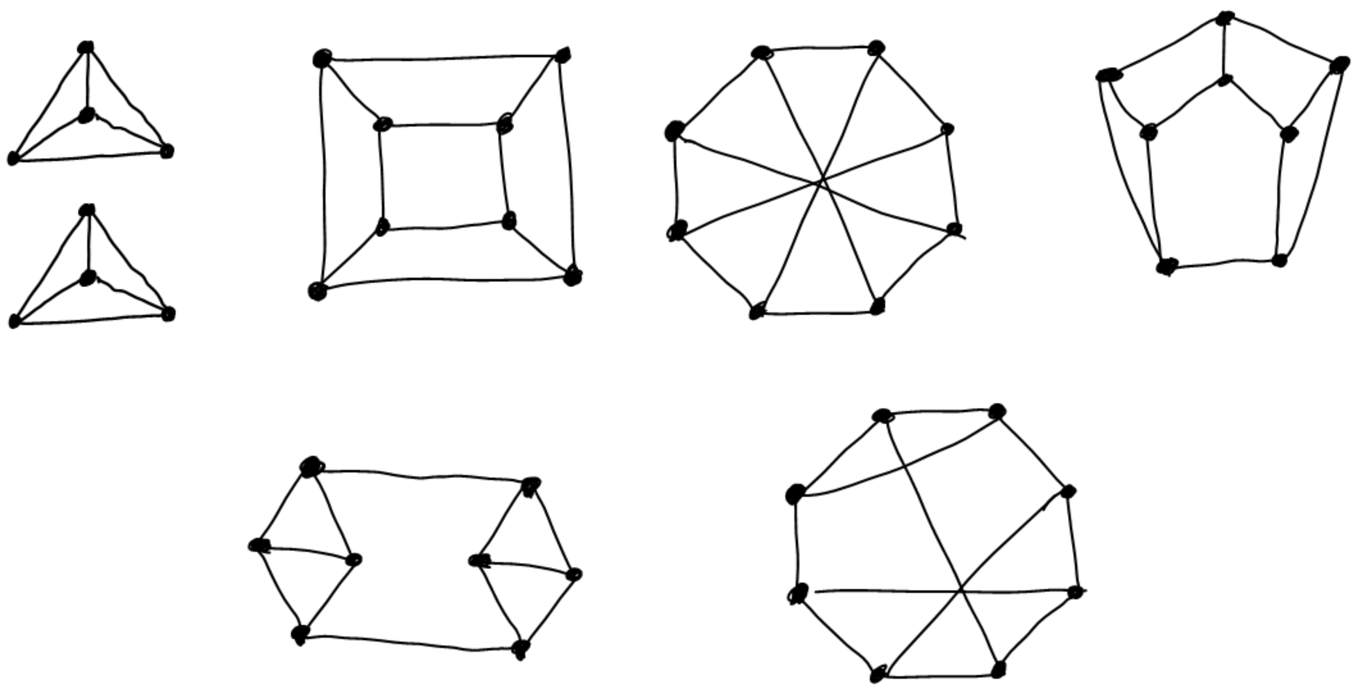
1.

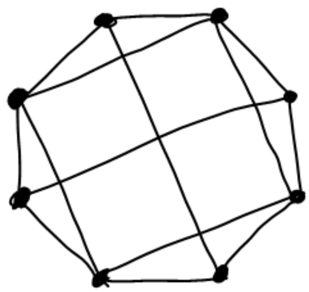
six vertices loopless

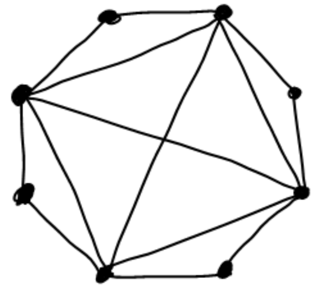


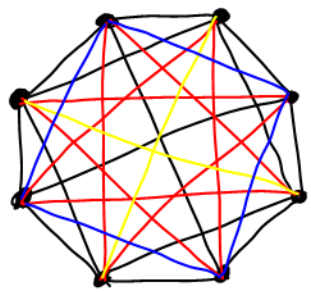
eight vertices, simple

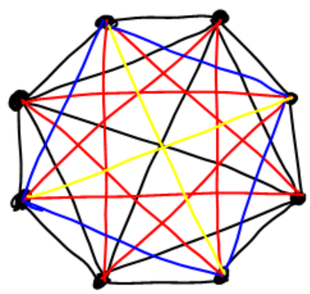


2.









(3) Let X = The number of degree-3 vertices
y = The number of degree-4 vertices.

Therefore $X+Y=n$ and

$3X+4Y=2(2n-2)$ by The handshaking Theorem.

So

$$\begin{array}{r} 4X+4Y=4n \\ -\ (3X+4Y=4n-4) \\ \hline X=4 \end{array}$$

(4) (a) Suppose $E(G) = E(C_1) \cup E(C_2) \cup \cdots \cup E(C_n)$ where the cycles are edge disjoint. Thus for any $v \in V(G)$, $d_G(v) = 2k$ in which $k \leq n$ is the number of cycles among $C_1, \ldots, C_n$ in which v appears. Thus $d_G(v)$ is even.

(b) Suppose every vertex of G has even degree. We will use induction on $|E(G)|$ to show that $E(G)$ is a disjoint union of edge sets of cycles.

Base Case $|E(G)| = 0$

In this case $E(G) = \phi$ is a disjoint union of edge sets of cycles in a vaccuous sense.

Induction Step

Suppose that for any graph G where every vertex has even degree and $|E(G)| \in \{0, \ldots, n\}$ that $E(G)$ is a disjoint union of edge sets of cycles.

Show the same is true when $|E(G)| = n+1$ and every vertex has even degree.

Since every vertex of G has even degree, we have that there is a cycle $C \subseteq G$. This was proven in class. Now let $G' = G - E(C)$. Note that

$$d_{G'}(v) = \begin{cases} d_G(v) & \text{if } v \notin V(C) \\ d_G(v)-2 & \text{if } v \in V(C) \end{cases}$$

In either case $d_{G'}(v)$ is even because $d_G(v)$ is even. Also $|E(G')| \leq n$ so by the induction hypothesis, $E(G')$ is a disjoint union of edge sets of cycles,

$$E(G') = E(C_1) \cup E(C_2) \cup \cdots \cup E(C_m).$$

Thus $E(G) = E(C_1) \cup E(C_2) \cup \cdots \cup E(C_m) \cup E(C)$ which is a disjoint union of edge sets of cycles. ∎