(1)

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

$$C_1 = \frac{1}{2}\binom{2}{1} = 1$$

$$C_2 = \frac{1}{3}\binom{4}{2} = \frac{1}{3}6 = 2$$

$$C_3 = \frac{1}{4}\binom{6}{3} = \frac{1}{\cancel{4}}\,\frac{\cancel{6}\cdot 5\cdot \cancel{4}}{\cancel{3!}} = 5$$

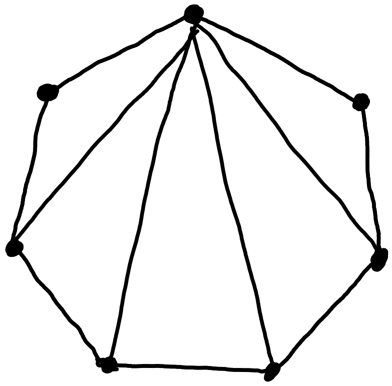
$$C_4 = \frac{1}{5}\binom{8}{4} = \frac{1}{\cancel{5}}\,\frac{\cancel{8}\cdot 7\cdot \overset{2}{\cancel{6}}\cdot \cancel{5}}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 14$$

$$C_5 = \frac{1}{6}\binom{10}{5} = \frac{1}{\cancel{6}}\,\frac{\cancel{10}\cdot \overset{3}{\cancel{9}}\cdot \overset{2}{\cancel{8}}\cdot 7\cdot \cancel{6}}{\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 42$$

$$C_6 = \frac{1}{7}\binom{12}{6} = 132$$

$$C_7 = \frac{1}{8}\binom{14}{7} = 429$$

2



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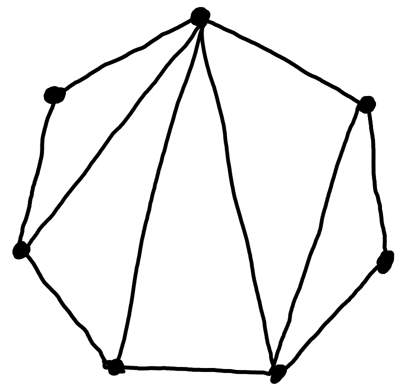

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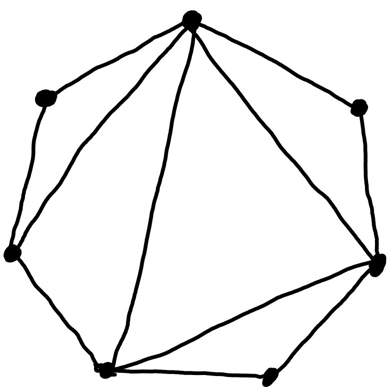

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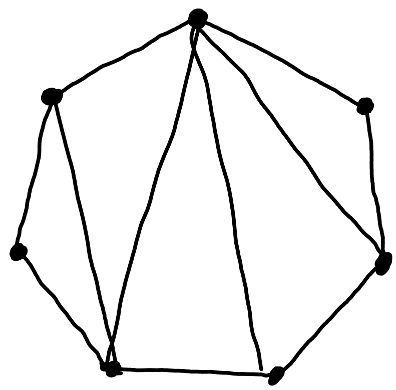

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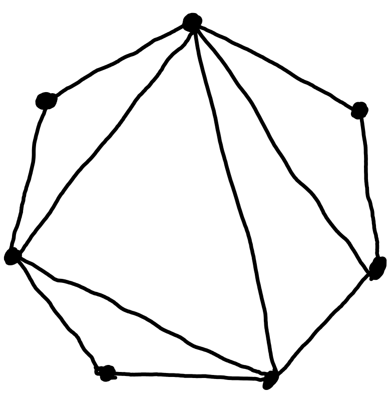

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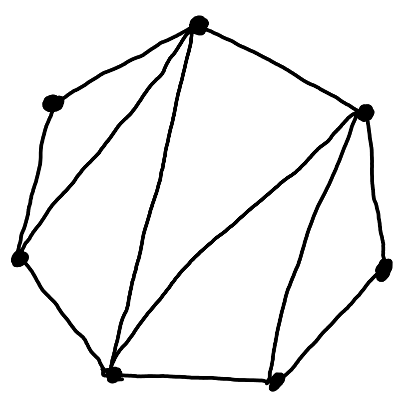
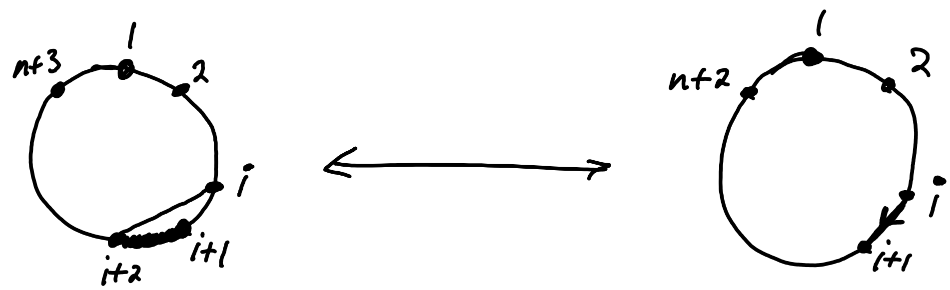


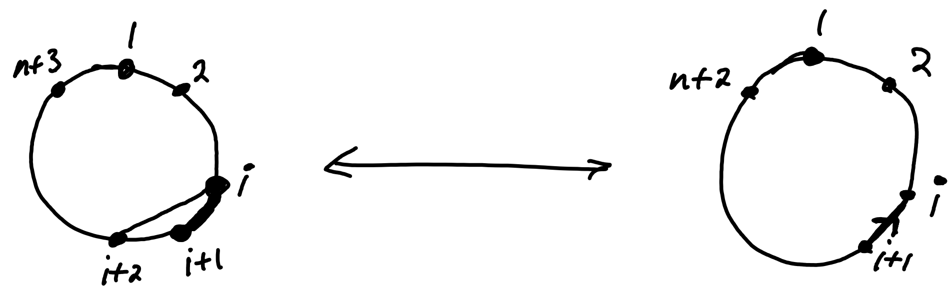
x7

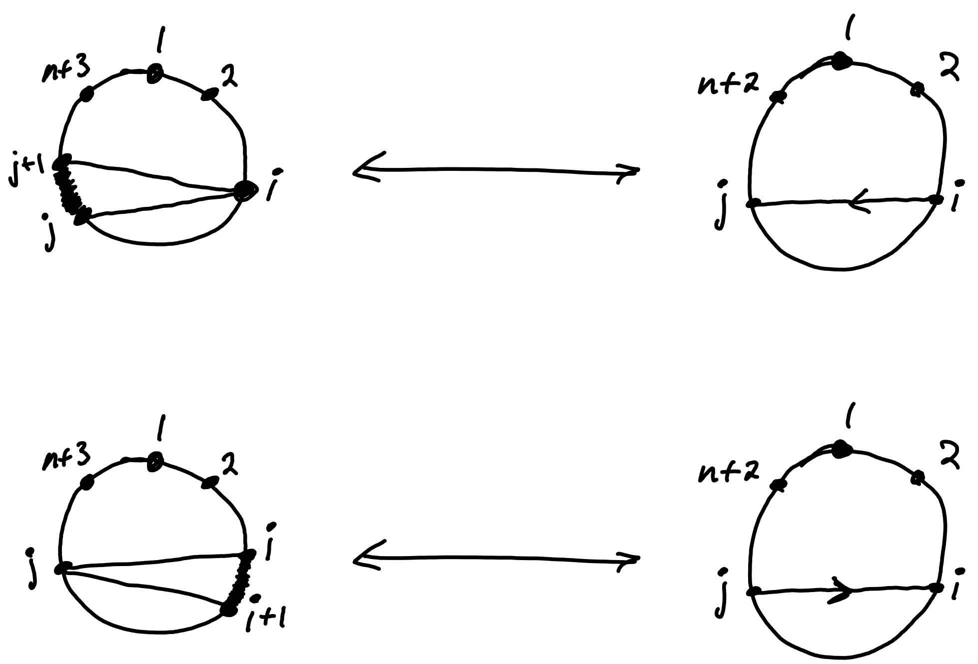
③ $(4n+2)C_n$ = The number of triangulated $(n+2)$-gons along with an arrow on one of its $2n+1$ edges.

$(n+2)C_{n+1}$ = The number of $(n+3)$-gons along with one of the outer edges colored black except for the edge $((n+3),1)$.

These two sets of objects correspond one-to-one via the following transformations.







Therefore $$(4n+2)C_n = (n+2)C_{n+1}$$ $\blacksquare$