Assignment #4

① The balls are the 52 numbers chosen from among $1, 2, \ldots, 99, 100$.

The baskets are

$\{1, 99\}$
$\{2, 98\}$
$\{3, 97\}$
$\vdots$
$\{49, 51\}$
$\{50\}$
$\{100\}$ which is 51 baskets in all.

Each ball is placed in its appropriate basket and so by the Pigeonhole Principle some basket gets 2 balls which, by the definition of the baskets, must add to 100.

A set of 51 numbers without a pair adding to 100 is

$1, 2, 3, \ldots\ldots, 49, 50, 100$

② Let g_i be the number of games played on day i.

Let
$$S_1 = g_1$$
$$S_2 = g_1 + g_2$$
$$S_3 = g_1 + g_2 + g_3$$
$$\vdots$$
$$S_{77} = g_1 + \cdots + g_{77}$$

Therefore $1 \leq S_1 < S_2 < \cdots < S_{77} = 131$

Now $23 < S_1 + 22 < S_2 + 22 < \cdots < S_{77} + 22 = 153$

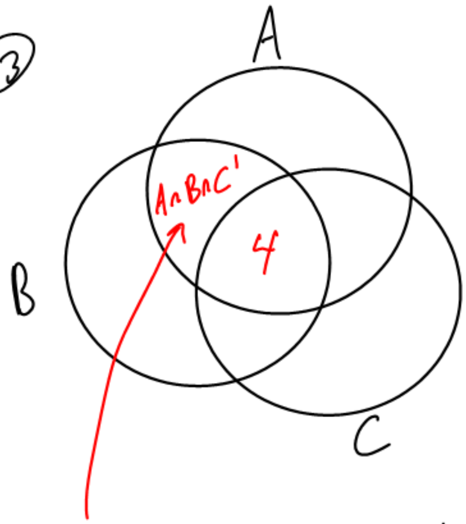
There are 154 numbers among $S_1, \ldots, S_{77}, S_1 + 22, \ldots, S_{77} + 22$ with only 153 possible values. Therefore two such numbers have the same value and it must be that
$$S_i = S_j + 22 \quad \text{for some } j < i$$

So
$$S_i - S_j = 22$$

So
$$g_{j+1} + g_{j+2} + \cdots + g_i = 22$$

So from day $j+1$ to day i exactly 22 games were played in total.

(3)



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$21 = 9 + 9 + 15 - 3 - 7 - 6 + |A \cap B \cap C|$$

$$4 = |A \cap B \cap C|$$

There is no non-negative answer possible for This cell.