

Assignment #2

Is due Wednesday 9/27.

It has been updated from
The initial version (Includes #12 page 155)

Assignment #3 Due Wednesday 10/4

Assignment #2 problem #4

Prove that
$$\sum_{k=0}^n (-1)^k \binom{n}{k}^2 = \begin{cases} 0 & \text{if } n \text{ is odd} \\ (-1)^m \binom{2m}{m} & \text{if } n=2m \end{cases}$$

If n is odd, then $\binom{n}{k} = \binom{n}{n-k}$ where k and $n-k$ have different parities (that is, one even and one odd).

Therefore you finish from here (one sentence)

If $n=2m$, Then consider the polynomials

$$(1-x^2)^n = ((1-x)(1+x))^n = (1-x)^n (1+x)^n$$

expand these three using the binomial theorem.

Then multiply the resulting polynomials $(1-x)^n (1+x)^n$

Since $(1-x^2)^n = (1-x)^n (1+x)^n$, The coefficients of x^n are the same. Compare to get the desired result.