Assignment #1 due Monday 9/18 at The beginning of class.

Write out your solutions on paper and give it to me at The beginning of class on 9/13.

Assignment #1 problem #3 Clarification

To count The number of straight flushes in The 54-card deck (The standard 52 + Joker R, Joker B)

A straight flush is 5 cards in a row all of The same suit.

A2345 45678 7---J 10JQKA.

23456 5---9 8---Q

34567 6---10 9---K

Straight flushes
w/ no Jokers

$10 \cdot 4 = 40$

Choices for kinds

Suit

Straight flushes
w/ one Joker

Pick one
of possible combinations
of kinds

8 9 10 J Q

Joker can't be here.

Joker could be one of these

BUT
with the highest

10 J Q K A

Joker could be anywhere

So here is how
to define the
straight flush
with one Joker.

Pick kinds – 10 choices

Pick suit – 4 choices

Pick Joker – 2 choices

Pick which kind the Joker represents – 4 choices
or
5 choices for
10, J, Q, K, A

Straight
flushes w/ two
Jokers.

game
idea.

Joker can't
be the
low card
except in
10, J, Q, K, A

In the
first 9 sequences
A, 2, 3, 4, 5
2, 3, 4, 5, 6,
⋮
9, 10, J, Q, K

There are $\binom{4}{2}$
placements
for the Joker.

In highest
10, J, Q, K, A there
are $\binom{5}{2}$.

Something like #4

Consider the 9-element multiset $\{3\cdot a, 3\cdot b, 3\cdot c\}$

$\{a, a, a, b, b, b, c, c, c\}$

How many 7-permutations are there?

That is, how many ways to list from left to right 7 of 9 elements?

List out all elements of

$\{1\cdot a, 3\cdot b, 3\cdot c\}$ in $\binom{7}{3\ 3\ 1}$ ways

or

$\{3\cdot a, 1\cdot b, 3\cdot c\}$ in $\binom{7}{3\ 3\ 1}$ ways

or

$\{3\cdot a, 3\cdot b, 1\cdot c\}$ in $\binom{7}{3\ 3\ 1}$ ways

or

$\{2\cdot a, 2\cdot b, 3\cdot c\}$ in $\binom{7}{3\ 2\ 2}$ ways

or

$\{2\cdot a, 3\cdot b, 2\cdot c\}$ in $\binom{7}{3\ 2\ 2}$ ways

or

$\{3\cdot a, 2\cdot b, 2\cdot c\}$ in $\binom{7}{3\ 2\ 2}$ ways

$$\frac{7!}{3!3!} = 140$$

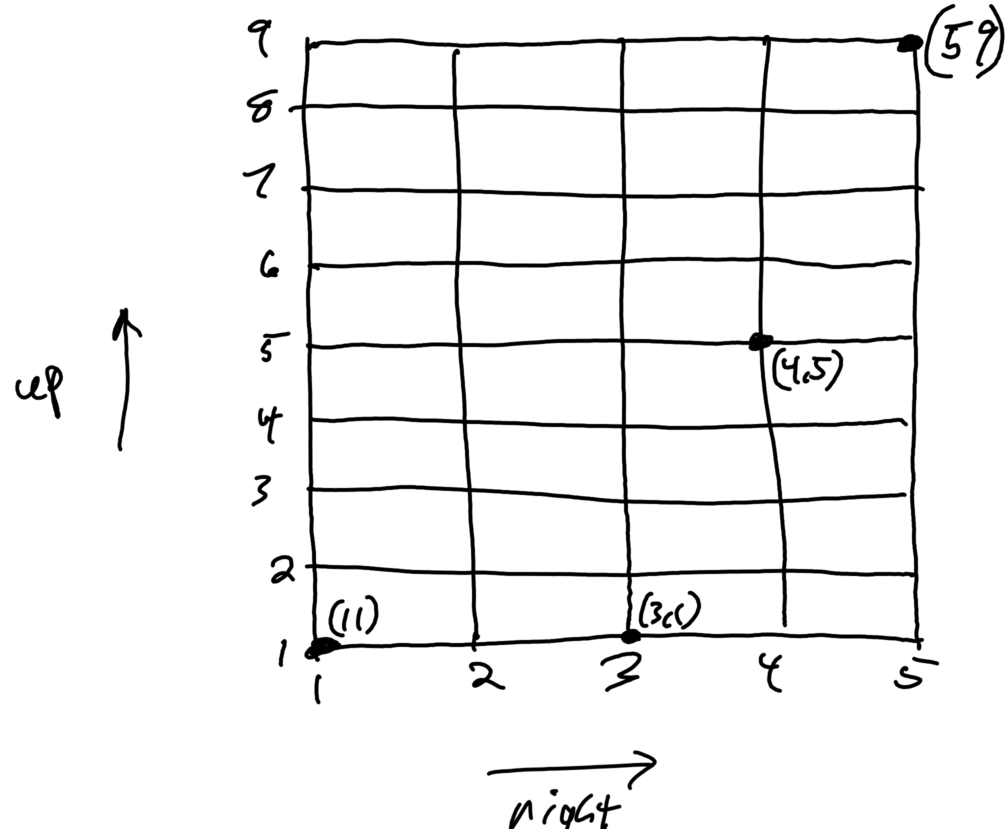
$$\frac{7!}{3!2!2!} = 210$$

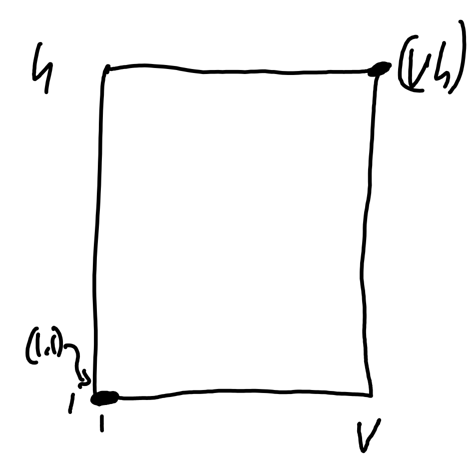
$$3(140) + 3(210) = 1050$$ 7-permutations

Another famous structure type counted using

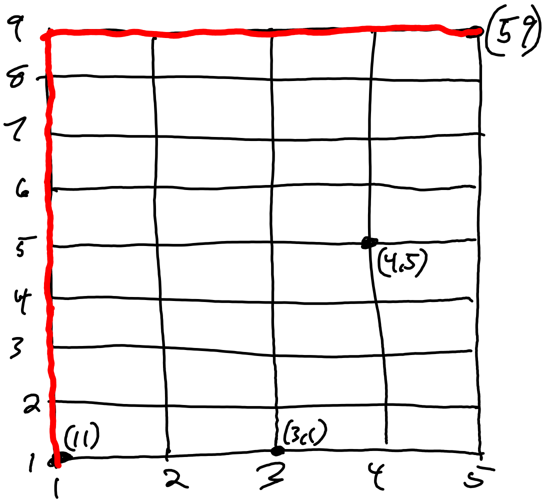
$$\binom{n+k-1}{n} = \binom{n+k-1}{k-1} =$$ the number of ways of placing n indistinguishable balls into k distinct baskets.

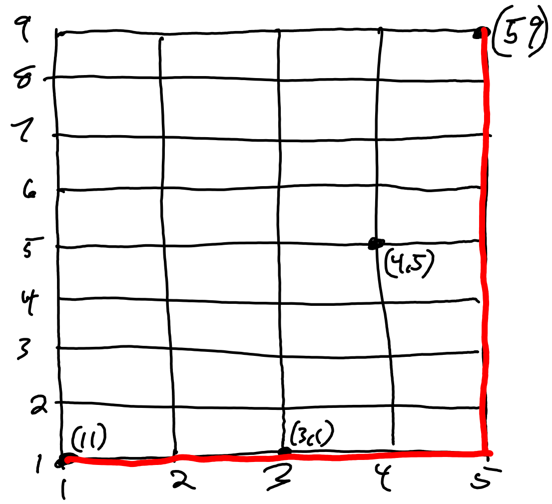
Consider a square grid with h horizontal lines and v vertical lines.

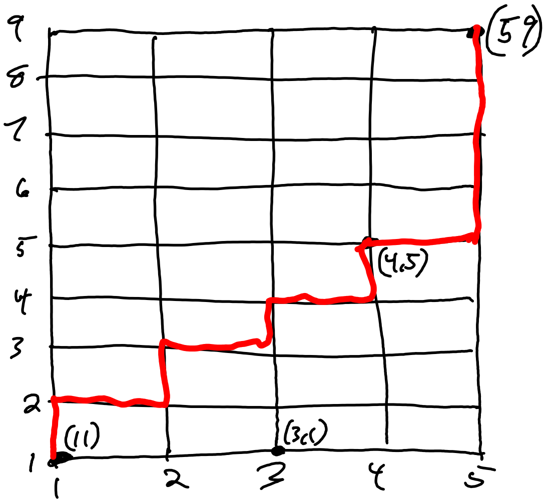


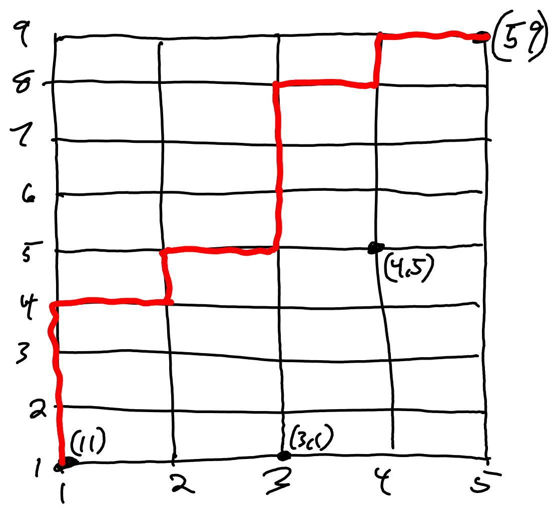


A <u>lattice</u> path from $(1,1)$ to (v,h) is a path along the grid having length $h+V-2$ where at each intersection we either go right one segment or up one segment.









how many such lattice paths are there??

For the 9×5 example we are distributing 8 identical choices of "up's" into 5 distinct vertical columns.

$\binom{8+5-1}{5-1}$ lattice paths.

More generally, for the grid with h horizontal and v vertical lines we are distributing $h-1$ identical "up's" into v distinct column lines.

This is done in

$$\binom{h-1+v-1}{v-1} = \binom{h+v-2}{v-1}$$