Homework Assignment #7
Due Date, Wednesday 11/01

(1) Let S and s denote the Stirling numbers of the second and first kind, respectively and say that $n \geq 2$.

a. Prove that $\mathsf{S}(n, n-2) = \binom{n}{3} + 3\binom{n}{4}$ using a combinatorial argument (not an algebraic argument or by mathematical induction).

b. Prove that $\mathsf{s}(n, n-2) = 2\binom{n}{3} + 3\binom{n}{4}$ using a combinatorial argument (not an algebraic argument or by mathematical induction).

(2) Recall the quotient rule from calculus

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{(\frac{d}{dx}f(x))g(x) - (\frac{d}{dx}g(x))f(x)}{(g(x))^2}$$

Given two sequences a_n and b_n find an expression for

$$\Delta\left(\frac{a_n}{b_n}\right)$$

that is reminiscent of the quotient rule.

(3) Find an expression for $\Delta^k(n2^n)$ as a closed-form algebraic expression in terms of n and k. (No proof is necessary, just a final expression.)

(4) Find c_3, c_2, c_1, c_0 such that

$$n^3 + n + 5 = c_3\binom{n}{3} + c_2\binom{n}{2} + c_1\binom{n}{1} + c_0\binom{n}{0}$$

Find an expression for

$$\sum_{k=0}^{n}(k^3 + k + 5)$$

(No proof is necessary, just calculations leading to the final expression in terms of binomial coefficients.)