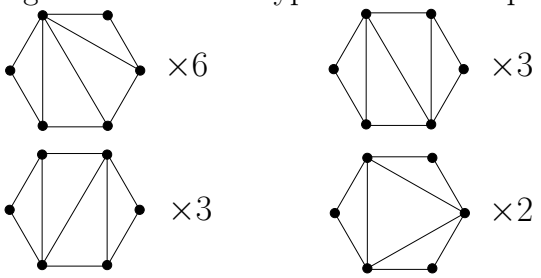
Homework Assignment #6
Due Wednesday 10/25

1. Let c_n denote the n^{th} Catalan number. Calculate $c_1, \ldots, c_7$.

2. We know that c_n equals the number of ways to triangulate a labeled, regular, $(n+2)$-gon by vertex chords and we know that $c_4 = 14$. Up to rotational symmetry, triangulations of an unlabeled hexagon are of the types shown where the multiplicities represent the number of labeled triangulations of each type. These multiplicities add to $c_4 = 14$.



Find all of the types and multiplicities of triangulations of the 7-gon. These multiplicities must add to $c_5 = 42$.

3. [redacted] Prove that $(4n+2)c_n = (n+2)c_{n+1}$. You will have to use the interpretation that c_n is the number of ways to triangulate a labeled $(n+2)$-gon. Note that there are $2n+1$ edges in an $(n+2)$-gon.