

**Homework Assignment #4**  
**Due Date, Wednesday 10/11**

1. Let  $\mathcal{L}$  be a collection of  $n$  lines in the plane that are in general position. Some of the regions in the plane defined by  $\mathcal{L}$  are bounded by polygons and some of the regions are unbounded. Use induction to prove the number of bounded regions is  $\frac{n^2 - 3n + 2}{2}$ .

2. Consider sequences of  $+1$ 's and  $-1$ 's containing an equal number of each character. Thus a sequence of length  $2n$  has  $n$   $+1$ 's and  $n$   $-1$ 's. Such a sequence is called *balanced* when each partial sum is non-negative. For example:  $+1, -1$  is balanced,  $+1, -1, +1, +1, -1, -1$  is balanced, and  $-1, +1$  is not balanced. Some properties of balanced sequences are as follows.

- If  $a$  is a balanced sequence, then  $+1, a, -1$  is a balanced sequence.
- If  $a$  and  $b$  are balanced sequences, then  $a, b$  is a balanced sequence.

Use induction on the length  $2n$  of a balanced sequence to prove that every balanced sequence of  $+1$ 's and  $-1$ 's can be constructed starting with  $+1, -1$  and then repeatedly using the two concatenation operations above.