Assignment #2

Problems from textbook 3.2, 3.6, 3.11, 3.16(i), 3.20, 3.23 cdef

Assignment #3

[A] Let $W = \text{Row}(A)$ be a subspace of $\mathbb{F}_5^7$

where

$$A = \begin{bmatrix} 1 & 4 & 3 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 & 3 & 4 \\ 3 & 0 & 4 & 0 & 3 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 1 & 1 & 0 \end{bmatrix}$$

* Find A basis for W.
* What is $\dim(W)$?
* What is $\dim(W^{\perp})$?
* Find a basis for $W^{\perp}$.

[B] Let $\vec{v} \in \mathbb{F}_2^n$. Prove that $\vec{v} \cdot \vec{v} = 0$ if and only if $wt(\vec{v})$ is even.

[C] Let $\vec{v} \in \mathbb{F}_3^n$. Prove that $\vec{v} \cdot \vec{v} = 0$ if and only if $wt(\vec{v})$ is a multiple of 3.

Exam 1 covers chapters 1, 2, 3, Chapter 4 Part 1. Assignments 1, 2, 3.

Scheduled for Wednesday February 8.

3.2 Addition and multiplication tables for $\mathbb{Z}_8$.

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

·	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

3.6

(a) Prove that $x^4+x^3+x^2+x+1$, x^4+x^3+1, x^4+x+1 in $\mathbb{F}_2[x]$ are irreducible.

First, a polynomial of degree n factors into degree 1 · degree $n-1$ if and only if it has a root.

$f(x)=x^4+x^3+x^2+x+1$ $f(x)=x^4+x^3+1$ $f(x)=x^4+x+1$

$f(0)=1$ $f(0)=1$ $f(0)=1$

$f(1)=5=1$ $f(1)=3=1$ $f(1)=3=1$

The only other possibility is degree-2 · degree-2.

The only irreducible polynomial of degree-2 in $\mathbb{F}_2$ is x^2+x+1

$$(x^2+x+1)(x^2+x+1) = x^4+(1+1)x^3+(1+1+1)x^2+(1+1)x+1$$
$$= x^4+x^2+1$$

which isn't any of the three polynomials above.

6. Show that x^2+1, x^2+x+2, x^2+2x+2 are irreducible in $\mathbb{Z}_3[x]$

These polynomials have degree-2 so they can be factored degree-1 · degree-1 if and only if they have roots.

$f(x) = x^2+1$	$f(x) = x^2+x+2$	$f(x) = x^2+2x+2$
$f(0) = 1$	$f(0) = 2$	$f(0) = 2$
$f(1) = 1+1 = 2$	$f(1) = 1+1+2 = 4 = 1$	$f(1) = 1+2+2 = 5 = 2$
$f(2) = 4+1 = 5 = 2$	$f(2) = 4+2+2 = 8 = 2$	$f(2) = 4+4+2 = 10 = 1$