

# Assignment #4 4.14, 4.31, A, B, C, D

A Let  $\vec{v} \in \mathbb{F}_2^n$ . Prove that  $\vec{v} \cdot \vec{v} = 0$  if and only if  $\text{wt}(\vec{v})$  is even.

B Let  $\vec{v} \in \mathbb{F}_3^n$ . Prove that  $\vec{v} \cdot \vec{v} = 0$  if and only if  $\text{wt}(\vec{v})$  is a multiple of 3.

C Let  $C$  be the  $[6, 3, 3]_2$ -linear code with generator matrix

$$G = \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array} \right)$$

① How many cosets does  $C$  have?

② Find a parity-check matrix  $H$  and calculate the syndromes for these cosets using as coset leaders vectors of smallest possible weight.

③ Decode the error words

011110, 101110, 111100  
using syndromes.

D Is the following matrix the generator matrix for a self-dual code? Explain.

$$G = \left( \begin{array}{c|cccccc} I_7 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{array} \right)$$