Assignment #6 due Friday 10/28 by 5pm

Exam 2 will cover chapters 4 and 5.

Scheduled for Monday 10/31.

For assignment #6 problem 2

You may use the fact that odd + odd = even

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Proposition: If $a, b \in \mathbb{Z}$, then

$a+b$ is even if and only if a and b have the same parity.

(2b) $[(13,0)] \underset{\substack{\text{by definition}\\ \text{of equivalence class}}}{=} \{(a,b) \mid (a,b) \sim (13,0)\} \overset{\text{by definition of } \sim.}{=} \{(a,b) \mid a^2+b^2 = 13^2\}$

So $[(13,0)]$ has elements $\underbrace{(\pm 13, 0)}_{\text{2 elements}}, \underbrace{(0, \pm 13)}_{\text{2 elements}}, \underbrace{(\pm 5, \pm 12)}_{\text{4 elements}},$

$\underbrace{(\pm 12, \pm 5)}_{\text{4 elements}}$

Note No explanation is necessary for the fact that $=$ in the usual sense is an equivalence relation: reflexive, symmetric, transitive.

Images ① Given $f: D \to R$ and $A \subseteq D$,

$$f(A) = \{f(x) \mid x \in A\}.$$

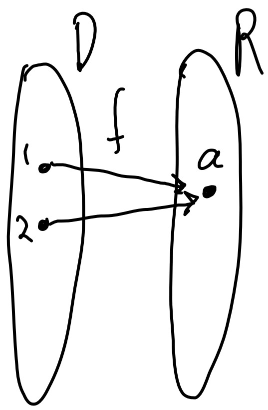
② If $x \in A$, then $f(x) \in f(A)$ by definition.
If $f(x) \notin f(A)$, then $x \notin A$ by contrapositive.
If $f(x) \in A$, it does not imply $x \in A$.

Consider $A, B \subseteq D$, $f(A \triangle B)$ and $f(A) \triangle f(B)$.

Let's prove that $f(A) \triangle f(B) \subseteq f(A \triangle B)$.

Let $y \in f(A) \triangle f(B)$. By definition of $\triangle$, $y \in f(A)$ or $y \in f(B)$ but not both. If $y \in f(A)$ but $y \notin f(B)$, then there is $x \in A$ but $x \notin B$ such that $y = f(x)$. By definition of $\triangle$, $x \in A \triangle B$. Therefore by definition of image $f(x) \notin f(A \triangle B)$. Thus $y \in f(A \triangle B)$. Similarly, if $y \in f(B)$ but $y \notin f(A)$, Then $y \in f(A \triangle B)$. $\blacksquare$

Give an example in which $f(A \triangle B) \nsubseteq f(A) \triangle f(B)$



Let $A = \{1\}$ and $B = \{2\}$.

So $A \triangle B = \{1, 2\}$ and $f(A \triangle B) = \{a\}$

and $f(A) = \{a\}$, $f(B) = \{a\}$, $f(A) \triangle f(B) = \emptyset$.

In This example $f(A \triangle B) \nsubseteq f(A) \triangle f(B)$.

Preimages ① Given $f: D \to R$ and $A \subseteq R$,

$$f^{-1}(A) = \{x \in D \mid f(x) \in A\}$$

② $x \in f^{-1}(A)$ if and only if $f(x) \in A$.

example Let's prove That $f^{-1}(A \triangle B) = f^{-1}(A) \triangle f^{-1}(B)$.

Let $x \in D$. Then

$x \in f^{-1}(A \triangle B)$, if and only if (def of preimage)

$f(x) \in A \triangle B$, if and only if (def of $\triangle$)

$f(x) \in A$ or $f(x) \in B$ but not both, if and only if (def of preimage)

$x \in f^{-1}(A)$ or $x \in f^{-1}(B)$ but not both, if and only if (def of $\triangle$)

$x \in f^{-1}(A) \triangle f^{-1}(B)$. $\blacksquare$