Exam #3 Wednesday 12/7 12:30-2:30 (in this room)

covering Chapters 6, 7 (Assignment 7, 8)

Assignment #8 due Monday 12/5 by 5pm.

Example to help with #1, aside from Example A in the notes

Let M and N be metric spaces with distance functions d_M and d_N. Define distance function d on $M \times N$ by $d((x,y),(a,b)) = d_M(x,a) + d_N(y,b)$. Prove that $M \times N$ is a metric space.

① Consider $(x,y) \in M \times N$.

So $d((x,y),(x,y)) = d_M(x,x) + d_N(y,y) = 0 + 0 = 0$

definition of d because M, N are metric spaces

② Consider $(x,y),(a,b) \in M \times N$ such that $(x,y) \neq (a,b)$. That is, $x \neq a$ or $y \neq b$. Because M, N are metric spaces $d_M(x,a) > 0$ or $d_N(y,b) > 0$. Therefore

$$d((a,b),(x,y)) = d_M(x,a) + d_N(y,b) > 0.$$

③ Consider $(a,b),(x,y) \in M \times N$.

$$d\big((a,b),(x,y)\big) = d_M(a,x) + d_N(b,y) \underset{\substack{\uparrow \\ \text{because} \\ M,N \text{ are} \\ \text{metric spaces}}}{=} d_M(x,a) + d_N(y,b) \underset{\substack{\uparrow \\ \text{definition} \\ \text{of } d}}{=} d\big((x,y),(a,b)\big).$$

④ Let $(a,b),(p,q),(x,y) \in M \times N$. So

$$d\big((a,b),(p,q)\big) + d\big((p,q),(x,y)\big) = d_M(a,p) + d_N(b,q) + d_M(p,x) + d_N(q,y)$$

$$= \big(d_M(a,p) + d_M(p,x)\big) + \big(d_N(b,q) + d_N(q,y)\big)$$

$$\geq d_M(a,x) + d_N(b,y) = d\big((a,b),(x,y)\big) \blacksquare$$