Homework Assignment #8 – Due Monday 12/5 by 5pm

1. Let M be a metric space with distance function d_M and N be a metric space with distance function d_N. Define a distance function on $M \times N$ by

$$d((a,b),(x,y)) = \max\Big\{d_M(a,x), d_N(b,y)\Big\}$$

Prove that $M \times N$ is a metric space with this distance function.

2. Let M be a metric space and $B_r(u)$ and $B_s(v)$ be open balls such that $B_r(u) \cap B_s(v) \neq \emptyset$.
   (a) Use the triangle inequality to show that $d(u,v) < r + s$.
   (b) Let $a \in B_r(u)$ and $b \in B_s(v)$. Use the triangle inequality to show that $d(a,b) < 2r + 2s$.

3. Let M be a metric space and a_n a sequence for which $a_n \to L$ and $a_n \to K$. Prove that $L = K$.

4. Let M, N, P be a metric spaces and $f\colon M \to N$ and $g\colon N \to P$ be continuous functions. Prove that $gf\colon M \to P$ is continuous.