Homework Assignment #6 – Due Wednesday 10/26

1. Define a relation on $\mathbb{Z} \times \mathbb{Z}$ by $(x, y) \sim (a, b)$ when $x^2 + y^2 = a^2 + b^2$.

(a) Prove that this is an equivalence relation.

(b) Find all of the elements of the following equivalence classes $[(0, 0)], [(1, 0)], [(2, 0)], [(5, 0)]$. No proof is necessary.

(c) Draw these equivalence classes in the xy-plane along with the familiar geometric objects which contain them.

2. Define a relation on $\mathbb{Z}$ by $x \equiv y$ when $x + y$ is even.

(a) Prove that this is an equivalence relation.

(b) What are the equivalence classes for this relation?

3. Given $a, b \in \mathbb{N}$, write $a|b$ to mean that there is $k \in \mathbb{N}$ such that $ak = b$. In other words when a *divides* b (equivalently, b is a *multiple* of a). Show that $|$ defines a partial order on $\mathbb{N}$.

4. Define a relation $\leq$ on $\mathbb{Z} \times \mathbb{Z}$ by $(a_1, b_1) \leq (a_2, b_2)$ when

$$a_1 \leq a_2 \text{ and } a_1 + b_1 \leq a_2 + b_2$$

Prove that $\leq$ is a partial order on $\mathbb{Z} \times \mathbb{Z}$.