Homework Assignment #5 – Due Wednesday 10/19 by 5pm

1. Given a function $f\colon D \to R$, let A and B be subsets of D.
   a. Prove that $f(A) - f(B) \subseteq f(A - B)$.
   b. Give an example showing that $f(A-B) \subseteq f(A) - f(B)$ is not necessarily true by providing a counterexample using $D = \{1, 2, 3, 4, 5\}$ and $R = \{a, b, c, d, e\}$.

2. Given a function $f\colon D \to R$, let $A \subseteq D$.
   a. Prove that $A \subseteq f^{-1}(f(A))$.
   b. Give an example with $f^{-1}(f(A)) \not\subseteq A$ using $D = \{1, 2, 3, 4, 5\}$ and $R = \{a, b, c, d, e\}$.
   c. Give an example with $f^{-1}(f(A)) = A$ using $D = \{1, 2, 3, 4, 5\}$ and $R = \{a, b, c, d, e\}$.

3. Define a function $f\colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ by $f\big((x, y)\big) = x + y$.
   a. Write down the elements of $f^{-1}\big(\{1, 2, 3, 4\}\big)$.
   b. Draw each of $f^{-1}\big(\{1\}\big)$, $f^{-1}\big(\{2\}\big)$, $f^{-1}\big(\{3\}\big)$, and $f^{-1}\big(\{4\}\big)$ on one standard xy-plane along with the familiar geometric objects which contain each of them.
   c. Is f surjective? Is f injective? (No proofs, just a yes or no answer.)

4. Let $D = \mathbb{R} - \{-2\}$, let $R = \mathbb{R} - \{2\}$, and let $f\colon D \to R$ be defined by

$$f(x) = \frac{2x + 5}{x + 2}$$

   a. Prove that f is a bijection.
   b. Find an algebraic expression for the inverse function $f^{-1}\colon R \to D$.

5. Consider functions $f\colon D \to R$ and $g\colon R \to P$ and let $A \subseteq P$. Prove that $(gf)^{-1}(A) = f^{-1}(g^{-1}(A))$.