Assignment #4 – Due Monday 9/26

1. Given the following three sets that are all subsets of universal set $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ find all of the following.

$A = \{4, 8\}$ $\quad$ $B = \{1, 3, 5, 7, 9\}$ $\quad$ $C = \{x \mid 3 \leq x < 9\}$

$A \cup B =$ $\{1,3,4,5,7,8,9\}$

$A \cap C =$ $\{4,8\}$

$B - C =$ $\{1,9\}$

$C' =$ $\{0,1,2,9\}$

$(A' \cup B) - C =$ $\{0,1,2,3,5,6,7,9\} - C = \{0,1,2,9\}$

$A \times C =$ $\{(4,3),(4,4),(4,5),(4,6),(4,7),(4,8),(8,3),(8,4),(8,5),(8,6),(8,7),(8,8)\}$

2. Let A, B, and C be sets. Using basic identities concerning sets prove that

$$(A \cup (A' \cap B))' = A' \cap B'$$

3. Let A, B, and C be sets. Prove that

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

by showing that $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ and also that $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$.

4. Let A and B be sets such that $A \subseteq B$. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

5. Let A and B be sets. Prove that $(B - A) \cup A = B$ if and only $A \subseteq B$ by proving that each of the following statements is true.

$$(B - A) \cup A = B \text{ implies that } A \subseteq B$$

$$A \subseteq B \text{ implies that } (B - A) \cup A = B$$

2. Let A, B, and C be sets. Using basic identities concerning sets prove that

$$(A \cup (A' \cap B))' = A' \cap B'$$

$(A \cup (A' \cap B))' \underset{\text{DeMorgans}}{=} A' \cap (A' \cap B)' \underset{\text{DeMorgan's}}{=} A' \cap (A \cup B') \underset{\text{Distribution}}{=}$

$(A' \cap A) \cup (A' \cap B') \underset{\text{Complementation and Identity}}{=} A' \cap B'$

3. Let A, B, and C be sets. Prove that

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

by showing that $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ and also that $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$.

Proof

Let $(x,y) \in A \times (B \cup C)$. By the definition of Cartesian Products, $x \in A$ and $y \in (B \cup C)$. By the definition of Unions, $y \in B$ or $y \in C$. Then, by the definition of Cartesian Products, $(x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$. Thus $(A \times B) \cup (A \times C)$ as required.

Proof

Let $(x,y) \in (A \times B) \cup (A \times C)$. By the definition of Unions, $(x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$. By the definition of Cartesian Products, $x \in A$ and $y \in B$, or $x \in A$ and $y \in C$. Then, by the definition of Unions, $y \in B \cup C$. Finally, by the definition of Cartesian Products, $(x,y) \in A \times (B \cup C)$, as required.

4. Let A and B be sets such that $A \subseteq B$. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

Proof

Let $A \subseteq B$. Consider some $x \in \mathcal{P}(A)$. By the definition of Power sets, $x \subseteq A$. Since $A \subseteq B$, $x \subseteq B$. Therefore, $x \in \mathcal{P}(B)$, as required.

5. Let A and B be sets. Prove that $(B-A)\cup A = B$ if and only $A \subseteq B$ by proving that each of the following statements is true.

$$(B-A)\cup A = B \text{ implies that } A \subseteq B$$

$$A \subseteq B \text{ implies that } (B-A)\cup A = B$$

Proof: Assume $A \not\subseteq B$. Then there is some $x \in A$, but $x \notin B$. By definition of Unions, then $(B-A)\cup A \neq B$, as required. ∎

$$A \subseteq B \text{ implies that } (B-A)\cup A = B$$

Proof: Assume $A \subseteq B$. We will first prove $(B-A)\cup A \subseteq B$, then $B \subseteq (B-A)\cup A$.

$\boxed{(B-A)\cup A \subseteq B}$ First let $x \in (B-A)\cup A$. By the definition of set subtraction, $x \in (B \cap A')\cup A$. Then, by Distribution, $x \in (B\cup A)\cap(A'\cup A)$. By complementation and identity properties, $x \in B\cup A$. By properties of Unions, since $A \subseteq B$, $B\cup A = B$, thus $x \in B$, as required. ∎

$\boxed{B \subseteq (B-A)\cup A}$ Next, let $x \in B$. Now, either $x \in A$ or $x \notin A$. If $x \in A$, then $x \in (B-A)\cup A$. If $x \notin A$, then by the definition of set subtraction $x \in B-A$. Therefore, $x \in (B-A)\cup A$. In either case we get the required outcome. ∎