Assignment ~~#2~~ #3 – Due date Monday 9/16

1. Translate the following English-language statements into predicate-logic statements. The domain is the entire world.

V(x)="x is a vegetable" Y(x)="x is a yellow" F(x)="x is a farmer" $G(x, y)$="x grows y"

(a) All vegetables are yellow.
(b) No vegetables are yellow.
(c) Only vegetables are yellow.
(d) Every farmer grows some kind of vegetable.
(e) No farmers grow vegetables.
(f) Some farmers grow all kinds vegetables.

2. Fill in each square with T or F.

	$\{0, \pm1, \pm2, \pm3, \ldots\}$	$\{1, 2, 3, 4, 5, \ldots\}$	$\{-2, -1, 0, 1, 2, 3\}$
$(\forall x)(\forall y)(x + y \geq x)$	F	T	F
$(\forall x)(\exists y)(x + y < x)$	T	F	T
$(\forall x)(\exists y)(x + y = 0)$	T	F	F
$(\exists x)(\forall y)(x + y = 0)$	F	F	F
$(\exists x)(\forall y)(xy = 0)$	T	F	T
$(\forall x)[(x < 0) \rightarrow (\exists y)(x + y = 0)]$	T	T	T
$(\exists x)(\exists y)[(x \neq 0) \wedge (x + y = 0)]$	T	F	T
$(\forall x)(\forall y)[(x < y) \vee (y < x)]$	F	F	F
$(\forall x)(\forall y)[(x \neq y) \rightarrow ((x < y) \vee (y < x))]$	T	T	T

3. Write the negation of each statement in proper English.
(a) Some lions roar.
(b) All lions roar.
(c) Only lions roar.
(d) Some lions eat zebras.
(e) No lions eat zebras.

4. An abstract predicate-logic statement is *valid* when it is true over any interpretation. Show that the following statements are not valid by giving for each statement an interpretation in which it is false. Make sure that you use complete sentences to define your predicates.
(a) $(\forall x)(P(x) \rightarrow Q(x)) \wedge (\exists x)P(x) \rightarrow (\forall x)Q(x)$
(b) $(\exists x)P(x) \wedge (\exists x)Q(x) \rightarrow (\exists x)(P(x) \wedge Q(x))$

(1) a. $(\forall x)(V(x) \rightarrow Y(x))$

b. $(\forall x)(V(x) \rightarrow Y'(x))$

c. $(\forall x)(Y(x) \rightarrow V(x))$

d. $(\forall x)(\exists y)(F(x) \rightarrow V(y) \wedge G(x,y))$

e. $(\forall x)(\forall y)(F(x) \wedge V(y) \rightarrow G'(x,y))$ or $(\forall x)(\forall y)(F(x) \rightarrow (V(y) \rightarrow G'(x,y)))$

f. $(\exists x)(\forall y)(F(x) \wedge (V(y) \rightarrow G(x,y)))$

③ a. All lions do not roar.

b. Some lions do not roar.

c. Something roars which isn't a lion.

d. All lions don't eat zebras.

e. Some lions eat zebras.

④ a. $D = \mathbb{Z}$

$P(x)$ = "x is a multiple of 4"

$Q(x)$ = "x is even"

b. $D = \mathbb{Z}$

$P(x)$ = "x is even."

$Q(x)$ = "x is odd"