Discussion problems for next time

Sections 4.4, 4.5, 4.6 from the syllabus.

Exam 2 covers sections 4.1, 4.3–4.7

Revised Grading Policy

Lowest mid-term exam	10%
3 higher mid-term exams	22.5% each
Final exam	22.5%

Grading Scale unchanged

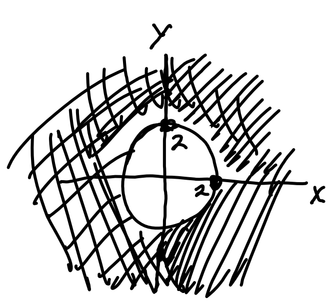
90–100	A
75–90	B
60–75	C
45–60	D
0–45	F

4.1 (6, 7) Find the domain

$f(x,y) = \sqrt{x^2+y^2-4}$

Need $x^2+y^2-4 \geq 0$

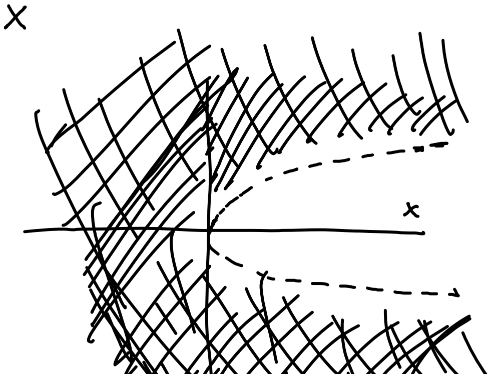
$x^2+y^2 \geq 4$



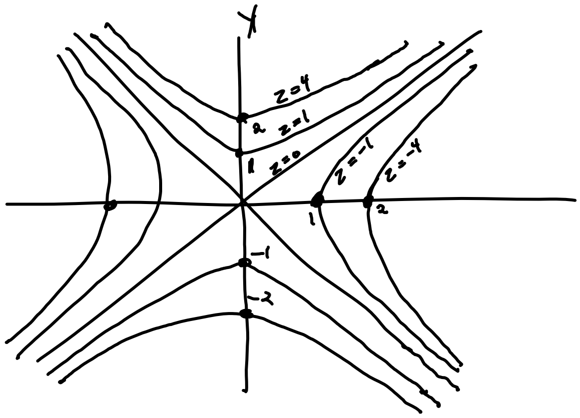
$f(x,y) = 4\ln(y^2-x)$

Need $y^2-x > 0$

$y^2 > x$



(14) $z = y^2 - x^2$ give contour map for $z = -4, -1, 0, 1, 4$



$0 = y^2 - x^2$

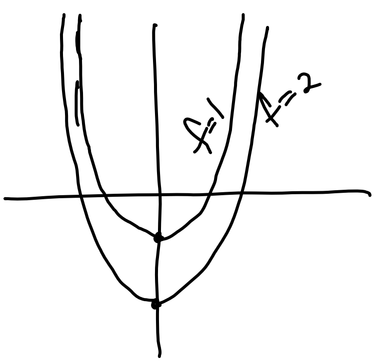
$0 = (y-x)(y+x)$

$y = x$ or $y = -x$

$1 = y^2 - x^2$

(16) example in class

(20) $f(x,y) = x^2 - y \quad f = 1, 2$



$\underline{f=1}$

$1 = x^2 - y$

$y = x^2 - 1$

(25) on your own

(30/31) omit

(47) Give a contour map w/ several level curves.

$$z = x^2 + y^2 - 2x - 2y$$

$\underline{z=0}$

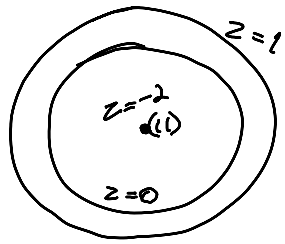
$0 = x^2 + y^2 - 2x - 2y$

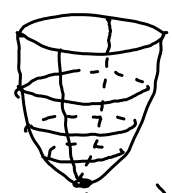
$2 = (x^2 - 2x + 1) + (y^2 - 2y + 1)$

$2 = (x-1)^2 + (y-1)^2$

$\underline{z=1}$

$3 = (x-1)^2 + (y-1)^2$





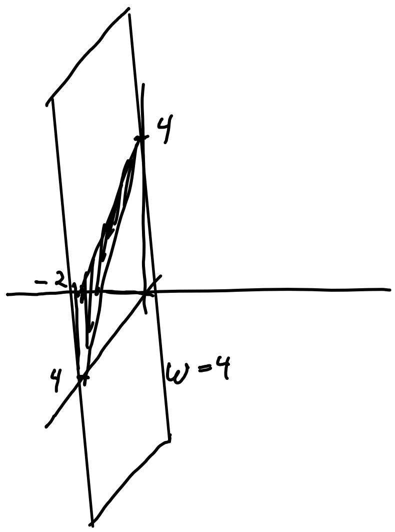
$(1,1,-2)$

$z = x^2 + y^2 - 2x - 2y$

(48) $w(x,y,z) = x - 2y + z$

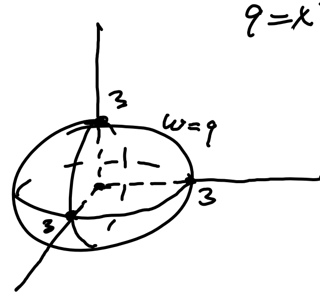
$w = 4$

$x - 2y + z = 4$



(49) $w(x,y,z) = x^2 + y^2 + z^2$ $\quad w = 9$

$9 = x^2 + y^2 + z^2$

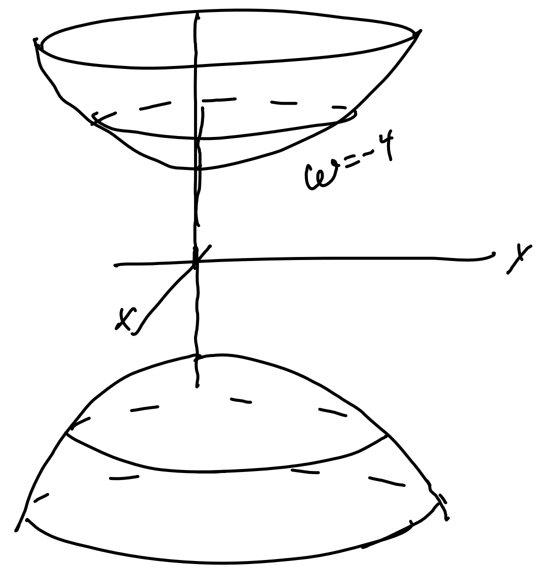


(50) $w = x^2 + y^2 - z^2$ $\quad w = -4$

$4 = z^2 - x^2 - y^2$

$z = k$ circles parallel to

$x = 0$ $\quad 4 = z^2 - y^2$



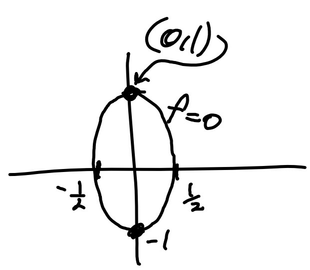
(53) $f(x,y) = 1 - 4x^2 - y^2$ find the equation of the level curve containing the point (0,1)

$f(0,1) = 1 - 0 - 1 = 0$

level curve for $f = 0$

$0 = 1 - 4x^2 - y^2$

$\boxed{4x^2 + y^2 = 1}$

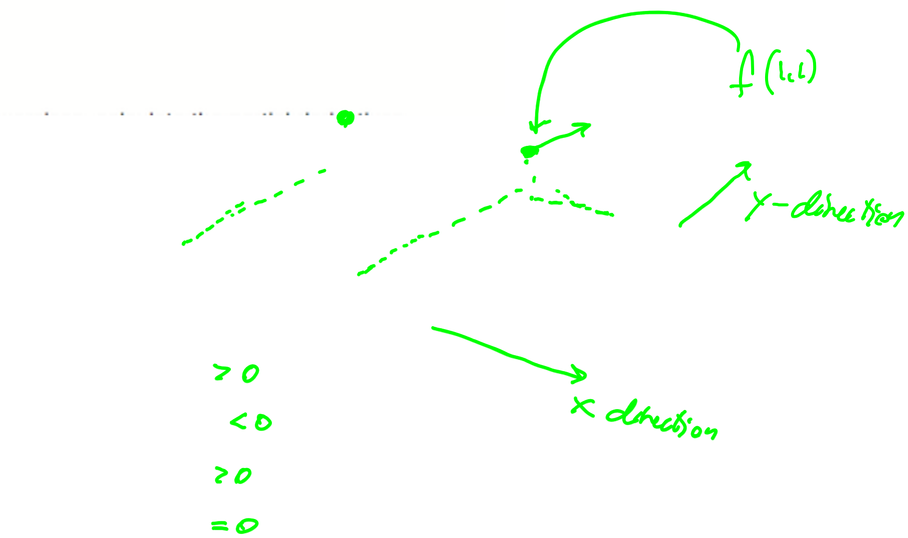


(54) $g(x,y) = y^2 \operatorname{Arctan}(x)$ find the equation of level curve containing $(1,2)$

$g(1,2) = 4 \operatorname{Arctan}(1) = \pi$

level curve is $\boxed{\pi = y^2 \operatorname{Arctan}(x)}$

$\boxed{\text{Sec 4.3}}$



(118) $z = \sin(3x)\cos(3y)$

$$\boxed{\frac{\partial z}{\partial x} = 3\cos(3x)\cos(3y)}$$

(122) $f = e^{xy}\cos(x)\sin(y)$

find f_y

$$\boxed{f_y = x e^{xy}\cos(x)\sin(y) + e^{xy}\cos(x)\cos(y)}$$

(124) $z = \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$

$$\boxed{\frac{\partial z}{\partial x} = \frac{1}{x} \qquad \frac{\partial z}{\partial y} = \frac{-1}{y}}$$

(128) $f(x,y) = \dfrac{xy}{x-y}$

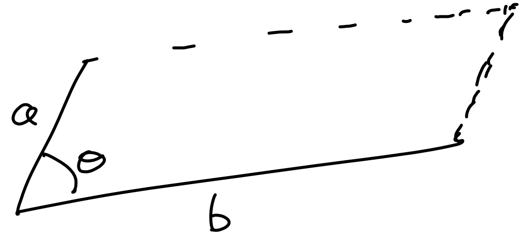
$$f_x(x,y) = \frac{y(x-y) - xy}{(x-y)^2} = \frac{\cancel{xy} - y^2 - \cancel{xy}}{(x-y)^2} = \frac{-y^2}{(x-y)^2}$$

$$f_x(2,-2) = \frac{-(-2)^2}{4^2} = \frac{-1}{4}$$

$$f_y(x,y) = \frac{x(x-y) + xy}{(x-y)^2} = \frac{x^2}{(x-y)^2}$$

$$f_y(2,-2) = \frac{4}{4^2} = \frac{1}{4}$$

(132) $A(a,b,\theta) = ab\sin(\theta)$



Find the rate of change of Area A with respect to a, b, and θ.

$$\frac{\partial A}{\partial a} = b\sin(\theta) \qquad \frac{\partial A}{\partial b} = a\sin(\theta) \qquad \frac{\partial A}{\partial \theta} = ab\cos(\theta)$$

(135) $z = \ln(x-y)$

find $z_{xy} = \frac{\partial}{\partial y}\frac{\partial}{\partial x} z = \frac{\partial^2 z}{(\partial y)(\partial x)}$

$$z_x = \frac{1}{x-y} = (x-y)^{-1} \qquad z_y = \frac{-1}{x-y} = -(x-y)^{-1}$$

$$z_{xy} = -(x-y)^{-2}(-1) = \frac{1}{(x-y)^2} \qquad z_{yx} = (x-y)^2(1) = \frac{1}{(x-y)^2}$$

(138) $z = e^x\tan(y)$

$$z_x = e^x\tan(y) \qquad z_y = e^x\sec^2(y)$$

$$z_{xy} = e^x\sec^2(y) \qquad z_{yx} = e^x\sec^2(y)$$ Same

(143) $f = x^2y^3z - 3xy^2z^3 + 5x^2z - y^3z$

will disappear at steps 2, 1

f_{xyz}

$f_x = 2xy^3z - 3y^2z^3 + 10xz$

$f_{xy} = 6xy^2z - 6yz^3$

$\boxed{f_{xyz} = 6xy^2 - 18yz^2}$

Show

(149) $z = e^x \sin(y)$ satisfies $z_{xx} + z_{yy} = 0$

$z_x = e^x \sin(y)$ $\quad z_y = e^x \cos(y)$

$z_{xx} = e^x \sin(y)$ $\quad z_{yy} = -e^x \sin(y)$

Thus

$z_{xx} + z_{yy} = 0$

(159) omit.