Discussion Problems for next time

4.1, 4.2 problems on syllabus
and answer questions on rest of 2.1-2.7, 3.1, 3.2, 3.4

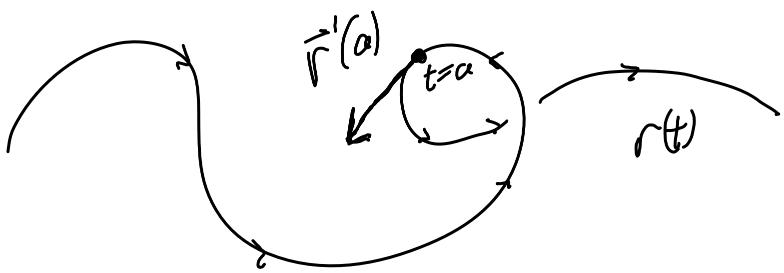
Exam 1 will cover sections 2.1-2.7 and 3.1, 3.2, 3.4
scheduled for This Wednesday 9/21.

Section 3.2

41) $\vec{r}(t) = \langle t^3, 3t^2, \frac{t^3}{6} \rangle$ so $\vec{r}\,'(t) = \langle 3t^2, 6t, \frac{t^2}{2} \rangle$

42) $\vec{r}(t) = \langle \sin(t), \cos(t), e^t \rangle$ so $\vec{r}\,'(t) = \langle \cos(t), -\sin(t), e^t \rangle$

52) $\vec{r}(t) = \langle 3t^3, 2t^2, \frac{1}{t} \rangle$ find tangent at $t=1$.



$\vec{r}\,'(t) = \langle 9t^2, 4t, \frac{-1}{t^2} \rangle$

at $t=1$ get tangent

$\boxed{\vec{r}\,'(t) = \langle 9, 4, -1 \rangle}$

(56) Unit tangent vector at $t=a$ is $\dfrac{\vec{r}\,'(a)}{|\vec{r}\,'(a)|}$

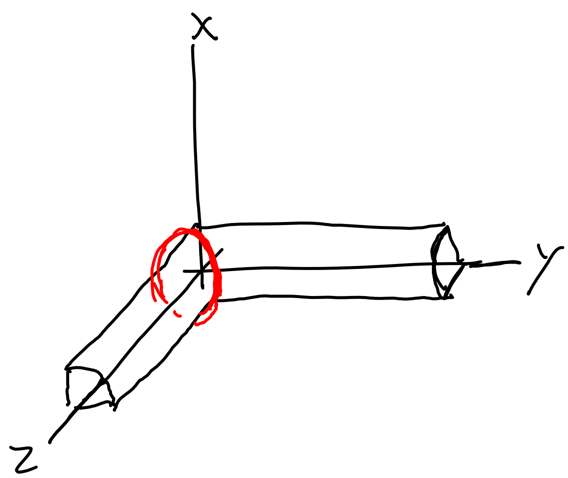
and of course, it has length $=1$.

The unit tangent function is $\dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \vec{u}(t)$.

$\vec{r}(t) = \langle \cos(t), \sin(t), \sin(t) \rangle$

$x^2+y^2=1$

$x^2+z^2=1$



$\vec{r}\,'(t) = \langle -\sin(t), \cos(t), \cos(t) \rangle$

$|\vec{r}\,'(t)| = \sqrt{\sin^2(t) + \cos^2(t) + \cos^2(t)}$

$= \sqrt{1+\cos^2(t)}$

$$\boxed{\vec{u}(t) = \left\langle \frac{-\sin(t)}{\sqrt{1+\cos^2(t)}}, \frac{\cos(t)}{\sqrt{1+\cos^2(t)}}, \frac{\cos(t)}{\sqrt{1+\cos^2(t)}} \right\rangle}$$

(63) $\vec{r}(t) = \langle -3t^5, 5t, 2t^2 \rangle$

$\vec{r}\,'(t) = \langle -15t^4, 5, 4t \rangle$

$\vec{r}\,''(t) = \langle -60t^3, 0, 4 \rangle$

$\vec{r}\,'(t) \cdot \vec{r}\,''(t) = (-15t^4)(-60t^3) + 0 + (4)(4t) = \boxed{900t^7 + 16t}$

(94) $\vec{r}(t) = \langle t, 2\sin(t), 2\cos(t) \rangle$ $\quad \vec{u}(t) = \langle \frac{1}{t}, 2\sin(t), 2\cos(t) \rangle$

$$\vec{r}(t) \times \vec{u}(t) = \begin{vmatrix} \hat{i} & \hat{j} & \vec{k} \\ t & 2\sin(t) & 2\cos(t) \\ \frac{1}{t} & 2\sin(t) & 2\cos(t) \end{vmatrix} = \left\langle 0, -2t\cos(t) + \frac{2}{t}\cos(t), 2t\sin(t) - \frac{2}{t}\sin(t) \right\rangle$$

(95) $\frac{d}{dt}\left(\vec{r}(t) \times \vec{u}(t)\right) =$

$$\left\langle 0, -2\cos(t) + 2t\sin(t) - \frac{2}{t^2}\cos(t) - \frac{2}{t}\sin(t), 2\sin(t) + 2t\cos(t) - \frac{2}{t^2}\sin(t) + \frac{2}{t}\cos(t) \right\rangle$$

(100)

$$\int \left\langle e^t, \sin(t), \frac{1}{2t-1} \right\rangle dt = \boxed{\left\langle e^t, -\cos(t), \frac{1}{2}\ln|2t-1| \right\rangle + \vec{C}}$$

(101)

$$\int_0^1 \left\langle t^{\frac{1}{3}}, \frac{1}{t+1}, e^{-t} \right\rangle dt = \left\langle \frac{3}{4}t^{\frac{4}{3}}, \ln|t+1|, -e^{-t} \right\rangle \Big]_{t=0}^{t=1}$$

$$= \left\langle \frac{3}{4}, \ln(2), \frac{-1}{e} \right\rangle - \langle 0, 0, -1 \rangle$$

$$\boxed{= \left\langle \frac{3}{4}, \ln(2), 1 - \frac{1}{e} \right\rangle}$$

(158)

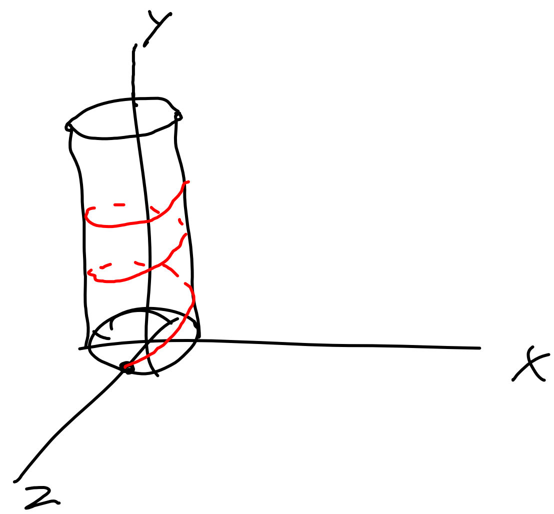
$\vec{r}(t) = \langle e^{-t}, t^2, \tan(t) \rangle$

$$v(t) = \vec{r}\,'(t) = \langle -e^{-t}, 2t, \sec^2(t) \rangle$$

$$\text{speed} = |\vec{r}\,'(t)| = \sqrt{e^{-2t} + 4t^2 + \sec^4(t)}$$

$$a(t) = r''(t) = \langle e^{-t}, 2, 2\sec^2(t)\tan(t) \rangle.$$

(162) $\vec{r}(t) = \langle \sin(t), t, \cos(t) \rangle$



$$\vec{r}\,'(t) = \vec{v}(t) = \langle \cos(t), 1, -\sin(t) \rangle$$

$$|\vec{r}\,'(t)| = \sqrt{\cos^2(t) + 1 + \sin^2(t)} = \sqrt{2}$$

$$\vec{r}\,''(t) = \vec{a}(t) = \langle -\sin(t), 0, -\cos(t) \rangle$$

(183) $\vec{a}(t) = \langle 0, t, t \rangle$ $\quad \vec{v}(1) = \langle 0, 5, 0 \rangle$ $\quad \vec{r}(1) = \langle 0, 0, 0 \rangle$

Find $\vec{r}(t)$.

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle 0, \tfrac{1}{2}t^2, \tfrac{1}{2}t^2 \rangle + \vec{C}$$

$$\langle 0, 5, 0 \rangle = \vec{v}(1) = \langle 0, \tfrac{1}{2}, \tfrac{1}{2} \rangle + C$$

$$\langle 0, \tfrac{9}{2}, -\tfrac{1}{2} \rangle = \vec{C}$$

$$\boxed{\vec{v}(t) = \left\langle 0,\ \tfrac{1}{2}t^2 + \tfrac{9}{2},\ \tfrac{1}{2}t^2 - \tfrac{1}{2} \right\rangle}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \left\langle 0,\ \tfrac{1}{6}t^3 + \tfrac{9}{2}t,\ \tfrac{1}{6}t^3 - \tfrac{1}{2}t \right\rangle + \vec{C}$$

$$\langle 0,0,0\rangle = \vec{r}(1) = \left\langle 0,\ \tfrac{14}{3},\ \tfrac{-1}{3} \right\rangle + \vec{C}$$

$$\left\langle 0,\ \tfrac{-14}{3},\ \tfrac{1}{3} \right\rangle = \vec{C}$$

$$\boxed{\vec{r}(t) = \left\langle 0,\ \tfrac{1}{6}t^3 + \tfrac{9}{2}t - \tfrac{14}{3},\ \tfrac{1}{6}t^3 - \tfrac{1}{2}t + \tfrac{1}{3} \right\rangle}$$

194 $\vec{a}(t) = \langle 1, e^t \rangle$, $\vec{v}(0) = \langle 0, 2\rangle$ $\vec{r}(0) = \langle 2, 0\rangle$

find $\vec{r}(t)$.

$$\vec{v}(t) = \langle t, e^t \rangle + \vec{C}$$

$$\langle 0, 2\rangle = \vec{v}(0) = \langle 0, 1\rangle + \vec{C}$$

$$\langle 0, 1\rangle = \vec{C}$$

$$\vec{v}(t) = \langle t, 1 + e^t\rangle$$

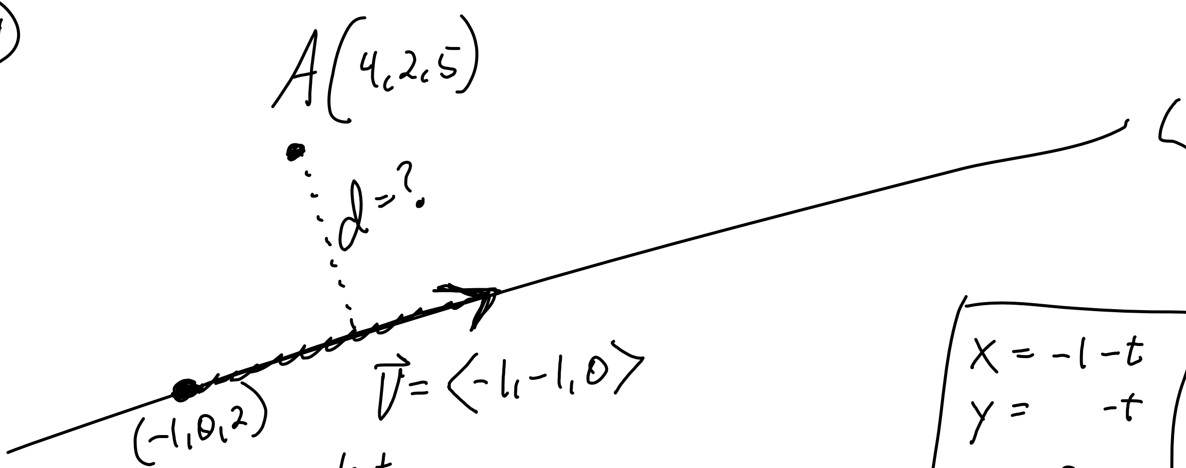
$$\vec{r}(t) = \left\langle \tfrac{1}{2}t^2,\ t + e^t \right\rangle + \vec{C}$$

$$\langle 2, 0\rangle = \vec{r}(0) = \langle 0, 1\rangle + \vec{C}$$

$\langle 2, -1 \rangle = \vec{c}$

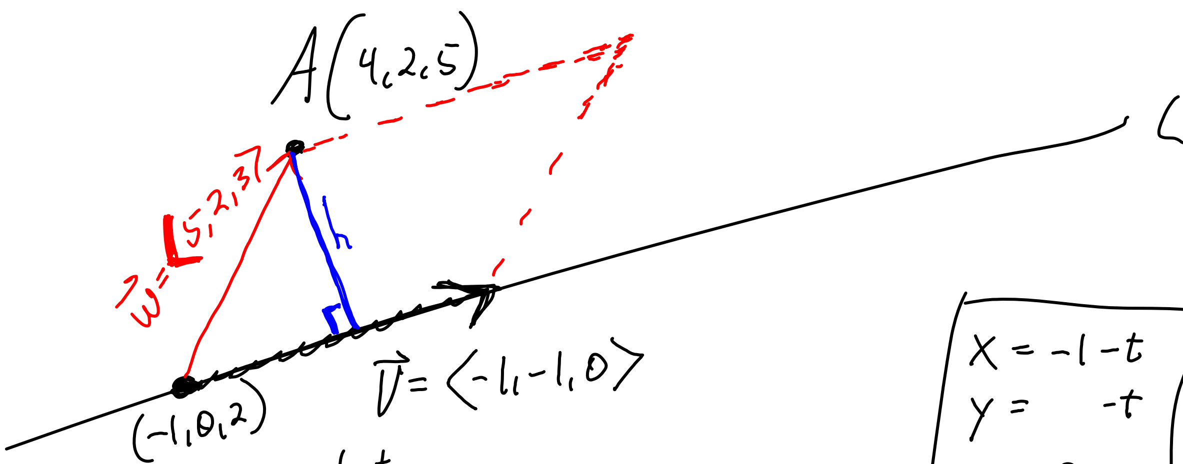
$$\boxed{\vec{r}(t) = \left\langle \tfrac{1}{2}t^2 + 2,\ t + e^t - 1 \right\rangle}$$

254



$$\boxed{\begin{aligned} x &= -1 - t \\ y &= -t \\ z &= 2 \end{aligned}}$$

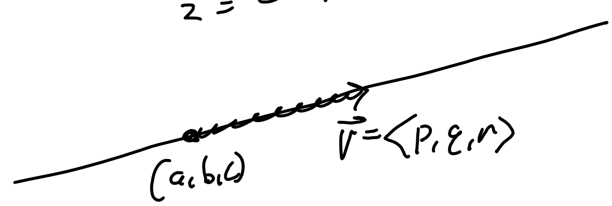
what is shortest distance of A to L



$$\boxed{\begin{aligned} x &= -1 - t \\ y &= -t \\ z &= 2 \end{aligned}}$$

what is shortest distance of A to L

$$x = a + pt$$
$$y = b + qt$$
$$z = c + rt$$



$$\text{Area} = \text{base} \cdot \text{height} = h\,|\vec{v}|$$

$$\text{Area} = |\vec{w} \times \vec{v}|$$

So $\quad h = \dfrac{|\vec{w} \times \vec{v}|}{|\vec{v}|}$

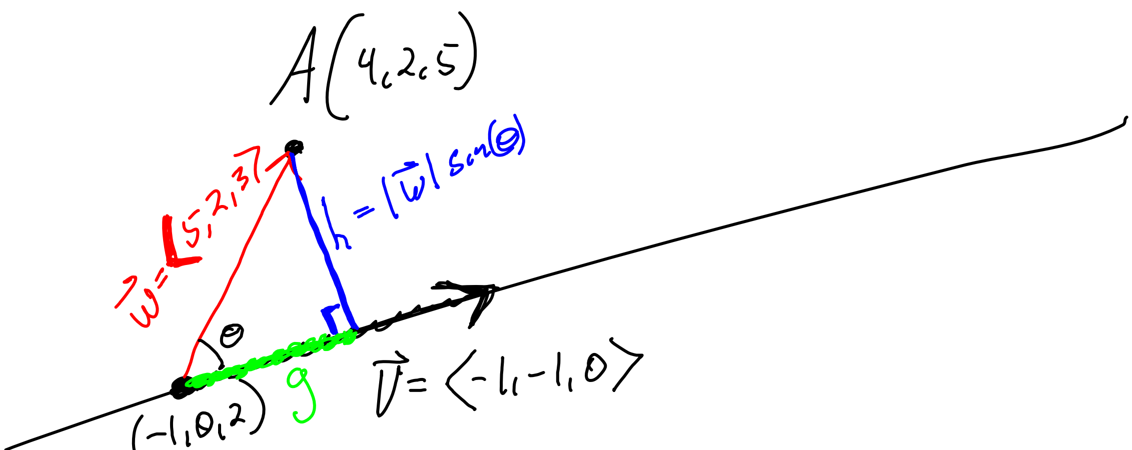
$$\vec{w} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 2 & 3 \\ -1 & -1 & 0 \end{vmatrix} = \langle 3, -3, -3 \rangle$$

$$|\vec{w} \times \vec{v}| = \sqrt{9+9+9} = 3\sqrt{3}$$

$$|\vec{v}| = \sqrt{1+1+0} = \sqrt{2}$$

distance $= \dfrac{3\sqrt{3}}{\sqrt{2}} = 3\sqrt{\dfrac{3}{2}}$.

A second method



green length $\quad g = \left||\vec{w}|\cos(\theta)\right| = \cancel{|\vec{w}|}\dfrac{\vec{v}\cdot\vec{w}}{|\vec{v}|\cancel{|\vec{w}|}} = \left|\dfrac{-7}{\sqrt{2}}\right| = \dfrac{7}{\sqrt{2}}$

$$|\vec{w}|^2 = g^2 + h^2$$

$$38 = \frac{49}{2} + h^2$$

$$\frac{76-49}{2} = h^2 \qquad \frac{27}{2} = h^2$$

$\boxed{\dfrac{3\sqrt{3}}{\sqrt{2}} = h}$ same answer.

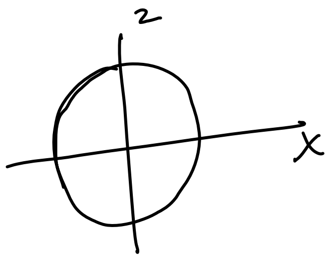
Sketch the surface given by the equation.

$$x^2+4y^2+z^2=1$$

let $y=k$ a constant

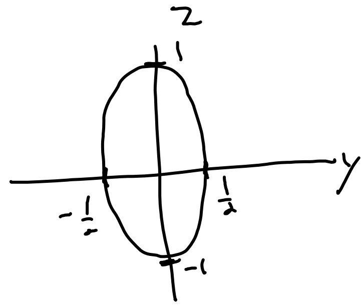
$$x^2+z^2=1-4k^2$$

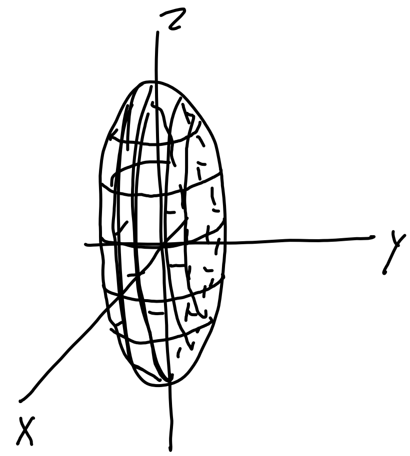
a circle parallel to the xz-plane



$x=0$

$$4y^2+z^2=1$$





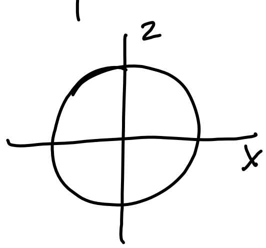
Sketch the surface given by

$$x^2-y^2+z^2=1$$

let $y=k$ a constant

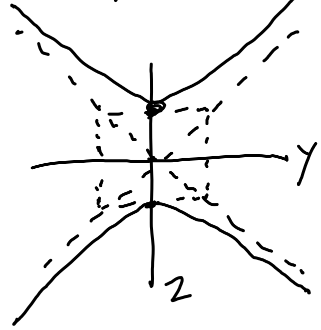
$$x^2+z^2=1+k^2$$

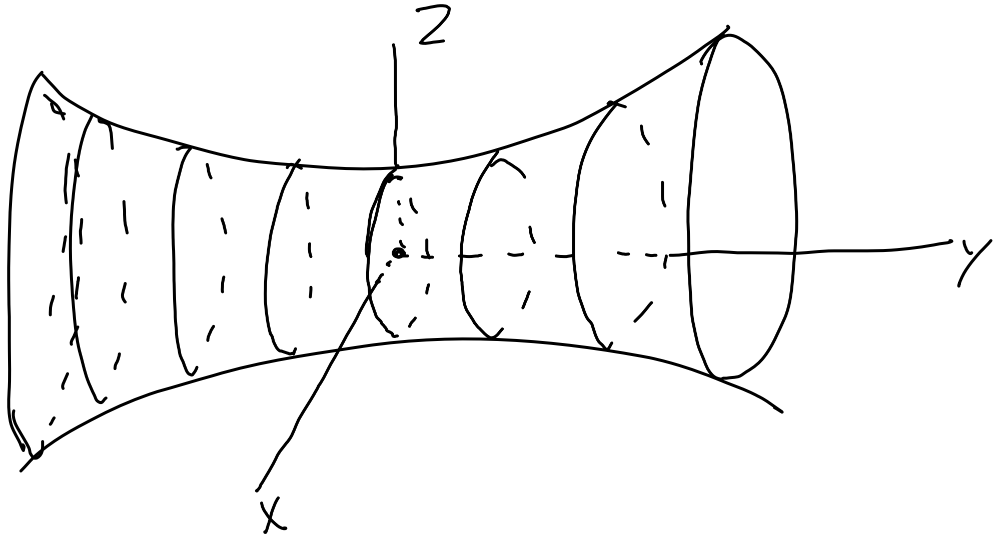
circle parallel to xz plane



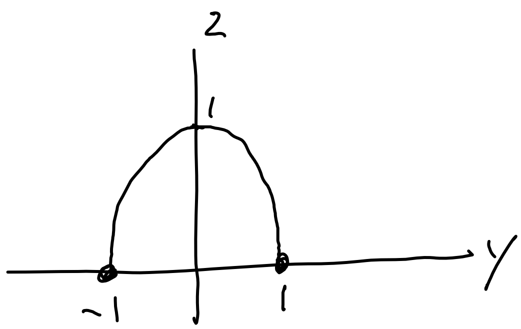
let $x=0$

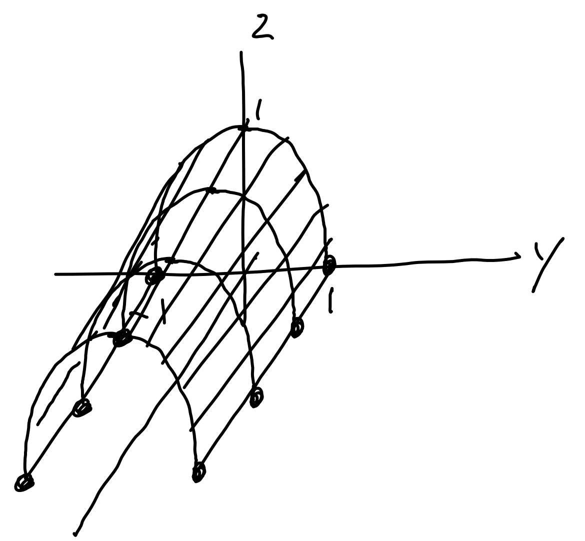
$$-y^2+z^2=1$$

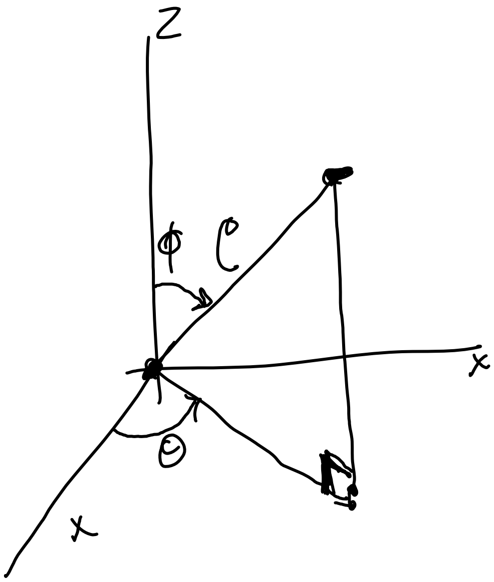




Sketch the surface given by $z = 1 - y^2,\ -1 \le y \le 1$







$(\rho, \theta, \varphi) \quad \rho \geq 0, \; 0 \leq \theta < 2\pi, \; 0 \leq \varphi \leq \pi$

$x = \rho\cos(\theta)\sin(\varphi)$

$y = \rho\sin(\theta)\sin(\varphi)$

$z = \rho\cos(\varphi)$

$x^2+y^2+z^2 = \rho^2$

(27)

(393) $\rho = 3$ same as $x^2+y^2+z^2=9$ which is the sphere of radius $=3$

(395) $\rho = 2\cos(\varphi)$ convert to rectangular coordinates

$\rho^2 = 2\rho\cos(\varphi)$

$x^2+y^2+z^2 = 2z$

$x^2+y^2+z^2-2z = 0$

$x^2+y^2+z^2-2z+1 = 1$

$x^2+y^2+(z-1)^2 = 1$

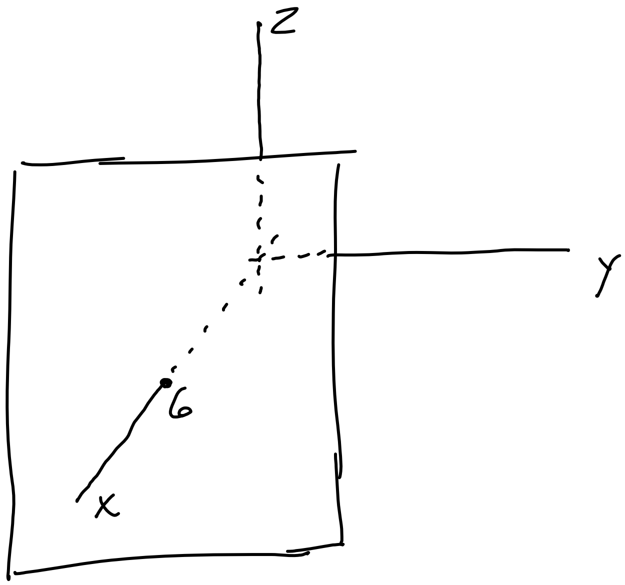
sphere centered at $(0,0,1)$

(398) $\rho = 6\csc(\varphi)\sec(\theta)$

$\rho = 6\,\frac{1}{\sin(\varphi)}\,\frac{1}{\cos(\theta)}$

$\rho\cos(\theta)\sin(\varphi) = 6$

$x = 6$



(388) $(\rho, \theta, \varphi) = \left(3, \frac{\pi}{4}, \frac{\pi}{6}\right)$ convert to (x, y, z)

$x = \rho\cos(\theta)\sin(\varphi) = 3\,\frac{\sqrt{2}}{2}\,\frac{1}{2} = \frac{3\sqrt{2}}{4}$

$y = \rho\sin(\theta)\sin(\varphi) = 3\,\frac{\sqrt{2}}{2}\,\frac{1}{2} = \frac{3\sqrt{2}}{4}$

$z = \rho\cos(\varphi) = 3\,\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

(392) $(-2, 2\sqrt{3}, 4) = (x, y, z)$ find (ρ, θ, φ)

$\rho^2 = x^2 + y^2 + z^2 = 4 + 12 + 16 = 32$

$\rho = 4\sqrt{2}$

$z = \rho\cos(\varphi)$

$4 = 4\sqrt{2}\cos(\varphi)$

$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = \cos(\varphi)$

$\boxed{\frac{\pi}{4} = \varphi}$

$x = \rho\cos(\theta)\sin(\varphi)$

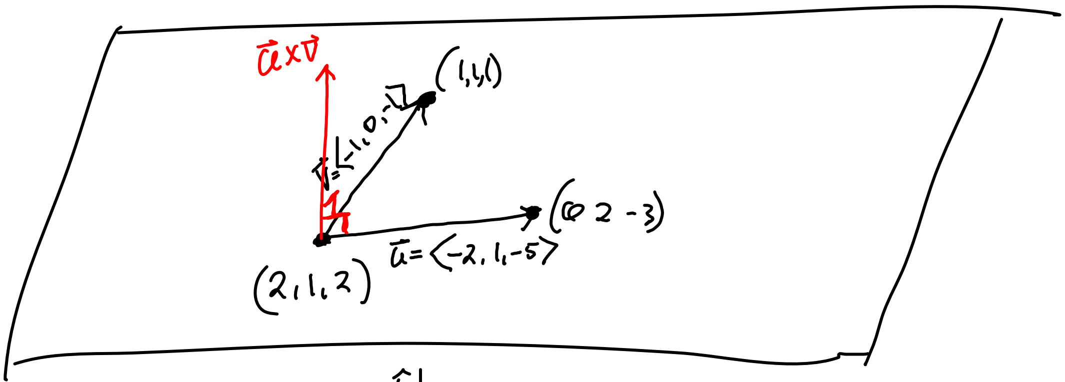
$-\cancel{2} = \cancel{4}\sqrt{2}\cos(\theta)\,\frac{\sqrt{2}}{\cancel{2}}$

$-1 = 2\cos(\theta)$

$-\frac{1}{2} = \cos(\theta)$

$\boxed{\frac{2\pi}{3} = \theta}$

Find the equation of the plane containing the points shown



$$\vec{n} = \vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 1 & -5 \\ -1 & 0 & -1 \end{vmatrix} = \langle -1, 3, 1\rangle$$

$$\vec{n}\cdot\langle x,y,z\rangle = \vec{n}\cdot\langle 1,1,1\rangle$$

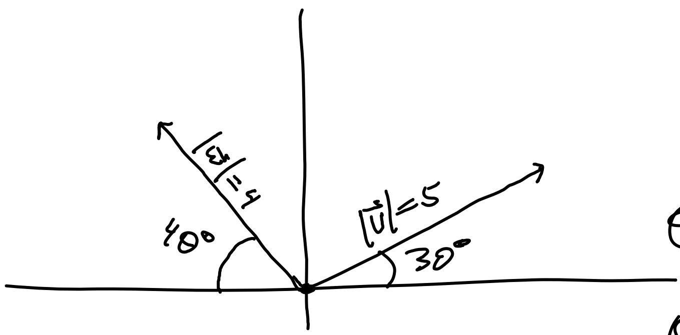
$$\boxed{-x+3y+z=3}$$

notice

$\vec{n}\cdot\langle 1\,1\,1\rangle = 3$

$\vec{n}\cdot\langle 0,2,-3\rangle = 3$

$\vec{n}\cdot\langle 2,1,2\rangle = 3$



(1) Find $\vec{v}+\vec{w}$ in component form. Round to 2 decimal places.

(2) Find $|\vec{v}+\vec{w}|$

(3) Find the angle of $\vec{v}+\vec{w}$ with the positive x-axis when the tail of $\vec{v}+\vec{w}$ is at the origin.

$$\vec{v} = \langle |\vec{v}|\cos(\theta), |\vec{v}|\sin(\theta)\rangle$$

$$= \langle 5\tfrac{\sqrt{3}}{2}, 5\tfrac{1}{2}\rangle$$

$$= \langle \tfrac{5}{2}\sqrt{3}, \tfrac{5}{2}\rangle = \langle 4.33, 2.5\rangle$$

$$\vec{w} = \langle -|\vec{w}|\cos(40), |w|\sin(40)\rangle$$

$$= \langle -3.064, 2.571\rangle$$

$$\vec{v}+\vec{w} = \langle 1.266, -.07\rangle$$

$$|\vec{v}+w| \approx 1.268$$

angle $\approx 0°$