Discussion Problems for next time

3.2, 3.4 problems on Syllabus

and answer questions on rest of 2.1-2.7, 3.1, 3.2, 3.4

Exam will cover sections 2.1-2.7 and 3.1, 3.2, 3.4

scheduled for 9/21.

Section 3.1

(5) $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$

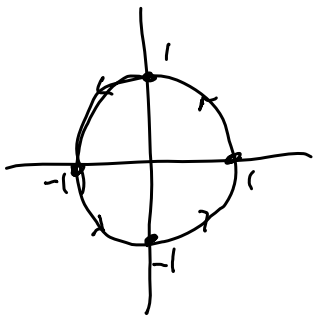
(a) $\lim_{t \to \frac{\pi}{3}} \vec{r}(t) = \left\langle \lim_{t \to \frac{\pi}{3}} \cos(t), \lim_{t \to \frac{\pi}{3}} \sin(t) \right\rangle = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

(b) $\vec{r}\left(\frac{\pi}{3}\right) = \left\langle \cos\left(\frac{\pi}{3}\right), \sin\left(\frac{\pi}{3}\right) \right\rangle = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

(c) Is $\vec{r}(t)$ continuous at $t = \frac{\pi}{3}$, yes because,

$$\lim_{t \to \frac{\pi}{3}} \vec{r}(t) = \vec{r}\left(\frac{\pi}{3}\right)$$

(d)

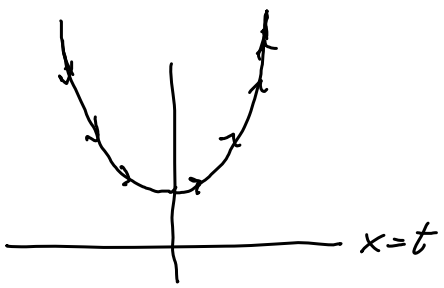


$x = \cos(t)$

$y = \sin(t)$

$x^2 + y^2 = 1$

(6) $\vec{r}(t) = \langle t, t^2+1 \rangle$ $\quad x = t$, $y = t^2+1$, $y = x^2+1$



(a) $\lim_{t\to -3} \vec{r}(t) = \langle \lim_{t\to -3} t, \lim_{t\to -3} t^2+1 \rangle = \langle -3, 10 \rangle$

(b) $\vec{r}(-3) = \langle -3, 10 \rangle$

(c) So $\vec{r}(t)$ is continuous at $t = -3$.

(d) $\vec{r}(t+2) - \vec{r}(t) = \langle t+2, (t+2)^2+1 \rangle - \langle t, t^2+1 \rangle$

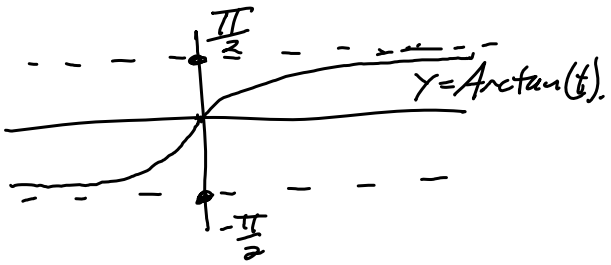
$= \langle t+2, t^2+4t+5 \rangle - \langle t, t^2+1 \rangle$

$= \langle 2, 4t+4 \rangle$

(9) on your own

(10) $\lim_{t\to\infty} \langle e^{-2t}, \frac{2t+3}{3t-1}, \arctan(2t) \rangle = \langle \lim_{t\to\infty} \frac{1}{e^{2t}}, \lim_{t\to\infty} \frac{2t+3}{3t-1}, \lim_{t\to\infty} \arctan(2t) \rangle$

$= \langle 0, \frac{2}{3}, \frac{\pi}{2} \rangle$



(16) $\vec{r}(t) = \langle t^2, \sqrt{t-3}, \frac{3}{2t+1} \rangle$ Domain = all $t \geq 3$ with $t \neq \frac{-1}{2}$

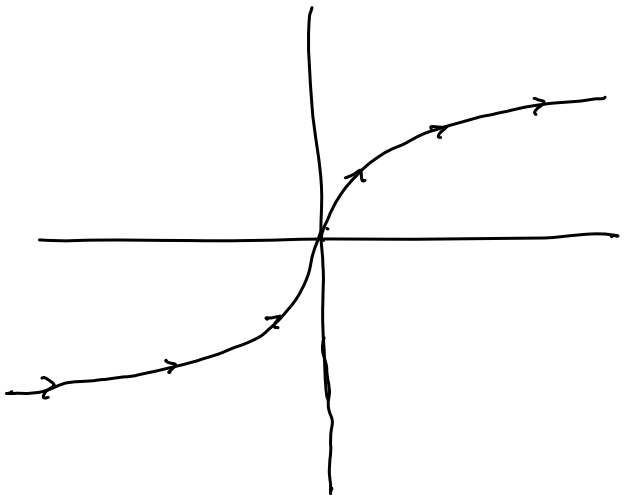
Domain = all $t \geq 3$

(23) $\vec{r}(t) = \langle t^3, 2t \rangle$

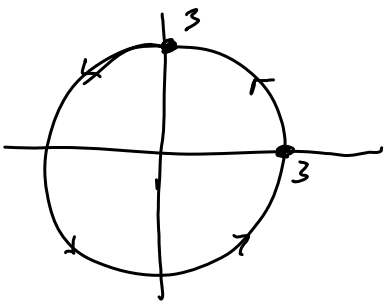
$x = t^3$

$y = 2t \longrightarrow \frac{y}{2} = t$

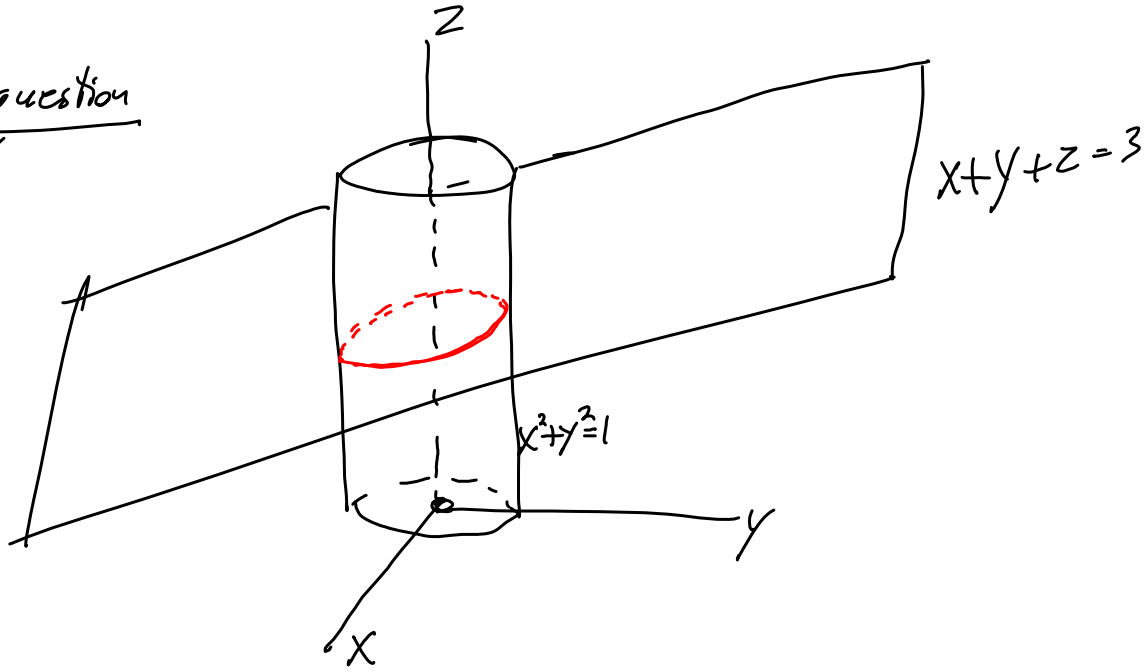
$x = \left(\frac{y}{2}\right)^3 = \frac{1}{8}y^3$



(26) $\vec{r}(t) = \langle 3\cos(t), 3\sin(t) \rangle$ $\quad x^2 + y^2 = 9\cos^2(t) + 9\sin^2(t) = 9(\cos^2(t) + \sin^2(t)) = 9$



Additional question



find a vector function $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ which traces out the intersection of the cylinder and plane given ??

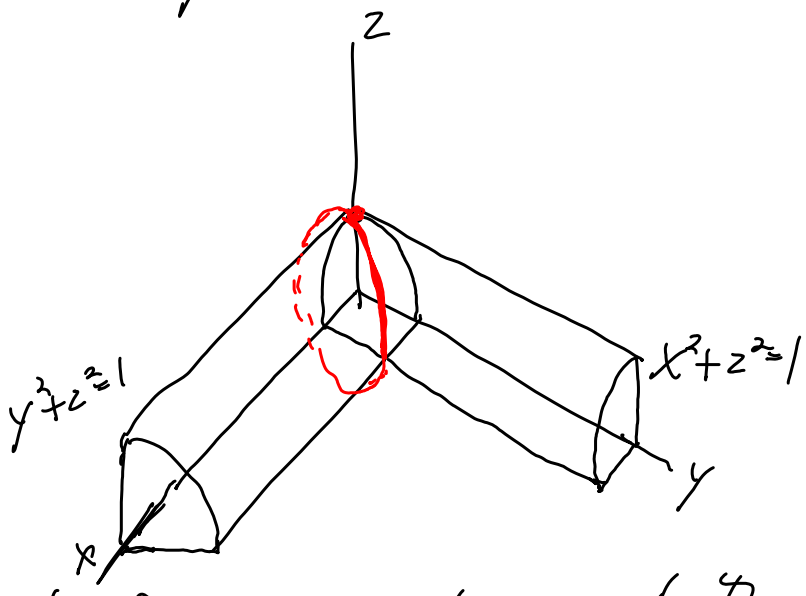
Since the curve lives on surface $x^2+y^2=1$ we can start with $x=\cos(t)$, $y=\sin(t)$.

Since the curve lives on $x+y+z=3$ also, we can set $z=3-x-y=3-\cos(t)-\sin(t)$

So $\vec{r}(t)=\langle \cos(t), \sin(t), 3-\cos(t)-\sin(t)\rangle$ lives on both surfaces and therefore the intersection of both surfaces.

For next time

Consider the intersection of the two cylinders $y^2+z^2=1$ and $x^2+z^2=1$.



Find a vector function which traces out this intersection.