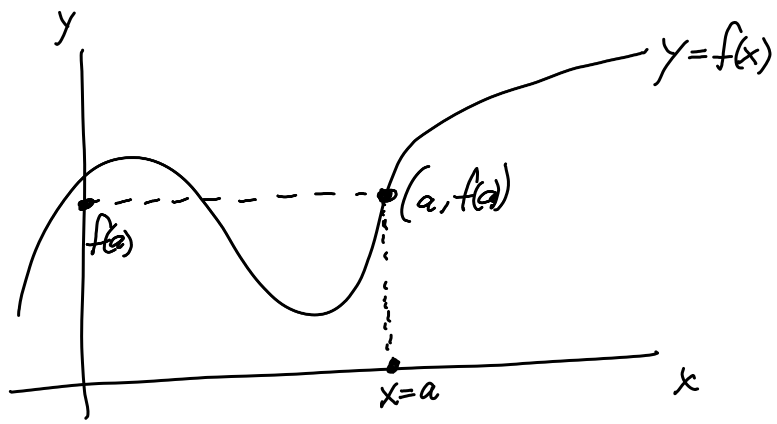
4.1 Functions of several variables.

Recall that the graph $y=f(x)$ for a function of one variable produces a curve in xy-plane satisfying the vertical-line test.



Consider a function $f(x,y)$ which uses 2 variables.

* the <u>domain</u> of $f(x,y)$ is the largest set of points (x,y) in the xy-plane for which $f(x,y)$ is defined.

<u>example</u>

What is the domain for $f(x,y)=\sqrt{(x+5)(y-1)}$

Need $(x+5)(y-1) \geq 0$

This means $x+5 \geq 0$ and $y-1 \geq 0$

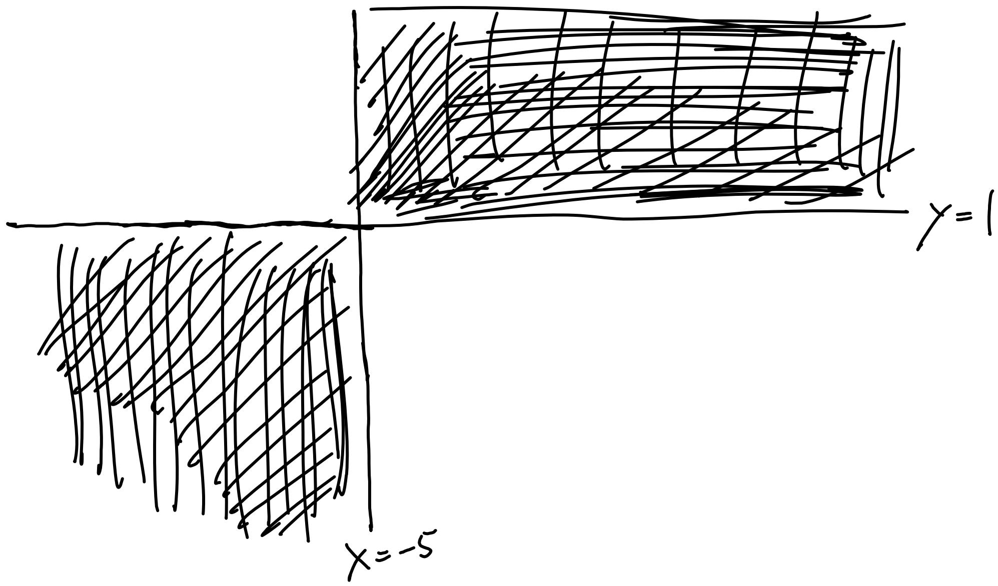
OR

$x+5 \leq 0$ and $y-1 \leq 0$

This means $x \geq -5$ and $y \geq 1$

OR

$x \leq -5$ and $y \leq 1$



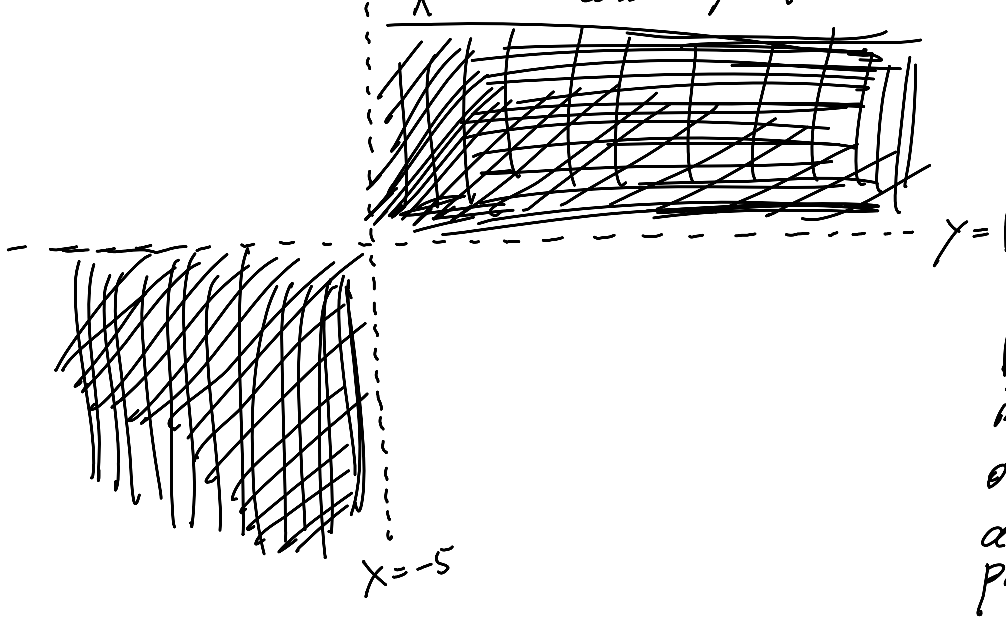
<u>example</u> Find the domain for $f(x,y) = \dfrac{1}{\sqrt{(x+5)(y-1)}}$

Here we need $(x+5)(y-1) > 0$ so $x+5 > 0$ and $y-1 > 0$
or
$x+5 < 0$ and $y-1 < 0$

This means $x > -5$ and $y > 1$

OR

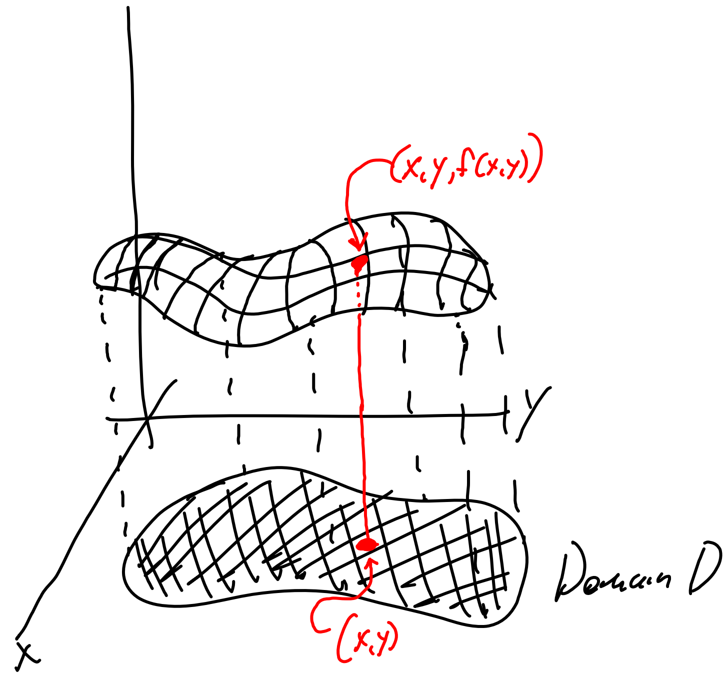
$x < -5$ and $y < 1$



Dashed lines indicate boundaries of the domain that are not actually part of the domain.

We can also consider the graph of 2-variable function $z = f(x,y)$ in 3-dimensional space.

The graph $z=f(x,y)$ also satisfies the vertical line test.

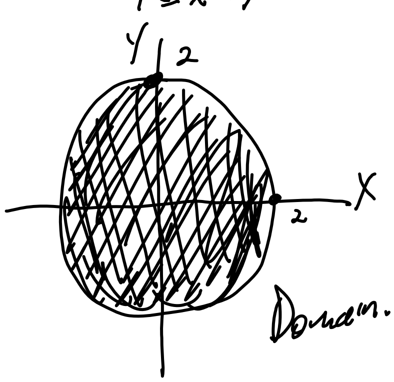


Example find the domain for $f(x,y)=\sqrt{4-x^2-y^2}$ and sketch the surface $z=\sqrt{4-x^2-y^2}$

<u>Need</u>

$4-x^2-y^2 \geq 0$

$4 \geq x^2+y^2$



<u>$z=k$</u>

$k=\sqrt{4-x^2-y^2}$

$x^2+y^2=4-k^2$

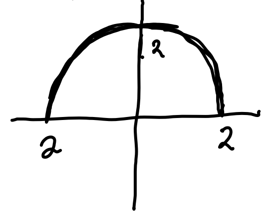
which is a circle parallel to xy-plane

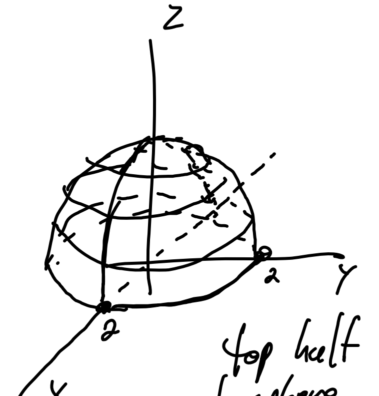
<u>$x=0$</u>

$z=\sqrt{4-y^2}$

$y^2+z^2=4$

top half of circle of radius 2





top half of of sphere of radius $=2$.

example Find the domain for $f(x,y)=\sqrt{y^2-x^2}$

Sketch the surface $z=f(x,y)$.

$y^2-x^2\ge 0$

$(y-x)(y+x)\ge 0$

$y-x\ge 0$ and $y+x\ge 0$

or

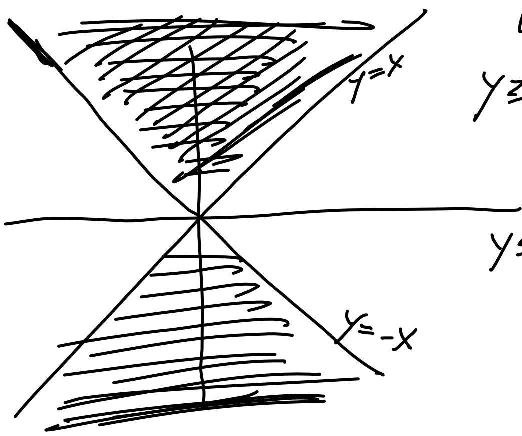
$y-x\le 0$ and $y+x\le 0$

which means

$y\ge x$ and $y\ge -x$

OR

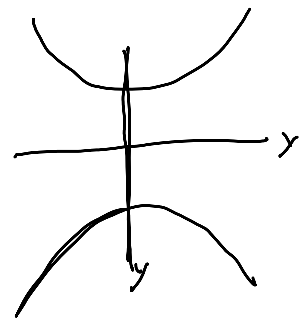
$y\le x$ and $y\le -x$



Let $z=k$

$k=\sqrt{y^2-x^2}$

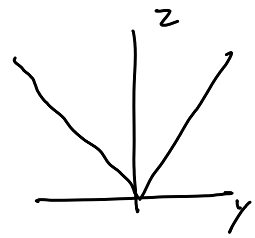
$k^2=y^2-x^2$

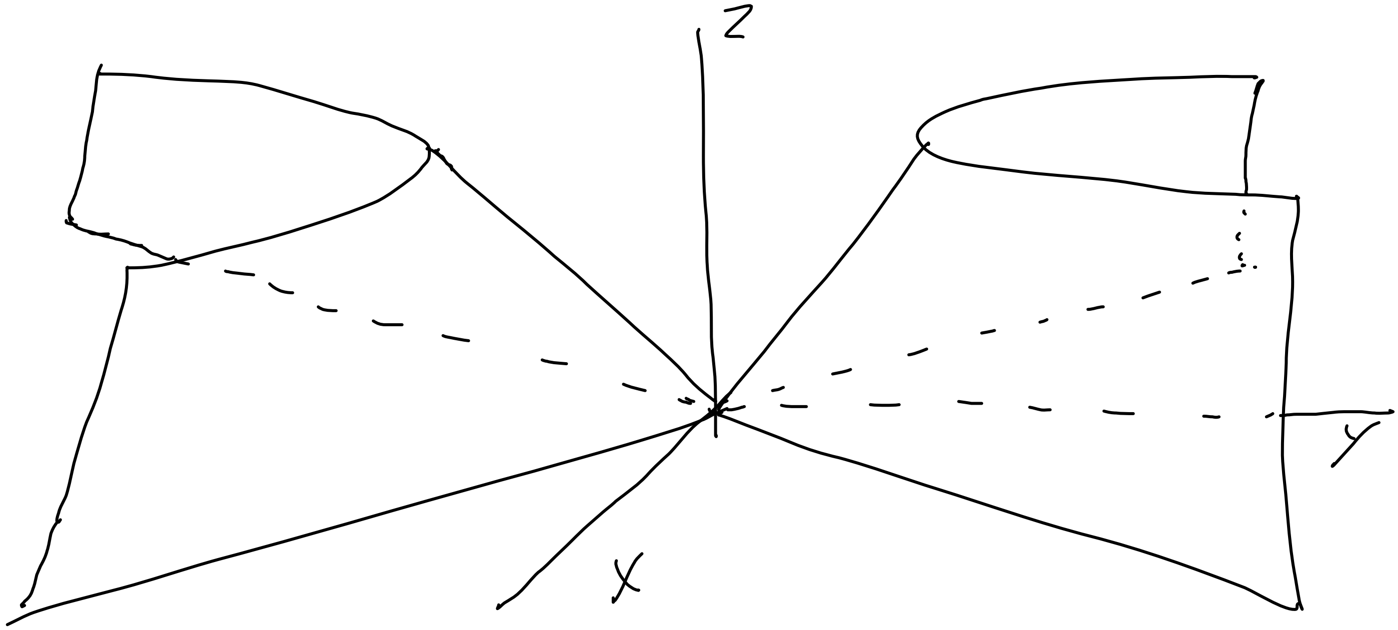


$x=0$

$z=\sqrt{y^2}$

$z=|y|$





Another way of looking at functions for 2 variables is with <u>level curves</u>. Different values of z will give different curves for x and y in the 2-dimensional plane. Several level curves drawn together is called a contour map for $z = f(x, y)$.

<u>example</u>

Make a contour map for $z = \sqrt{x^2+y^2}$ using values $z = 0, 1, 2, 3$

$z=0$ $\quad 0 = \sqrt{x^2+y^2}$

$0 = x^2+y^2$

$z=1$ $\quad 1 = \sqrt{x^2+y^2}$

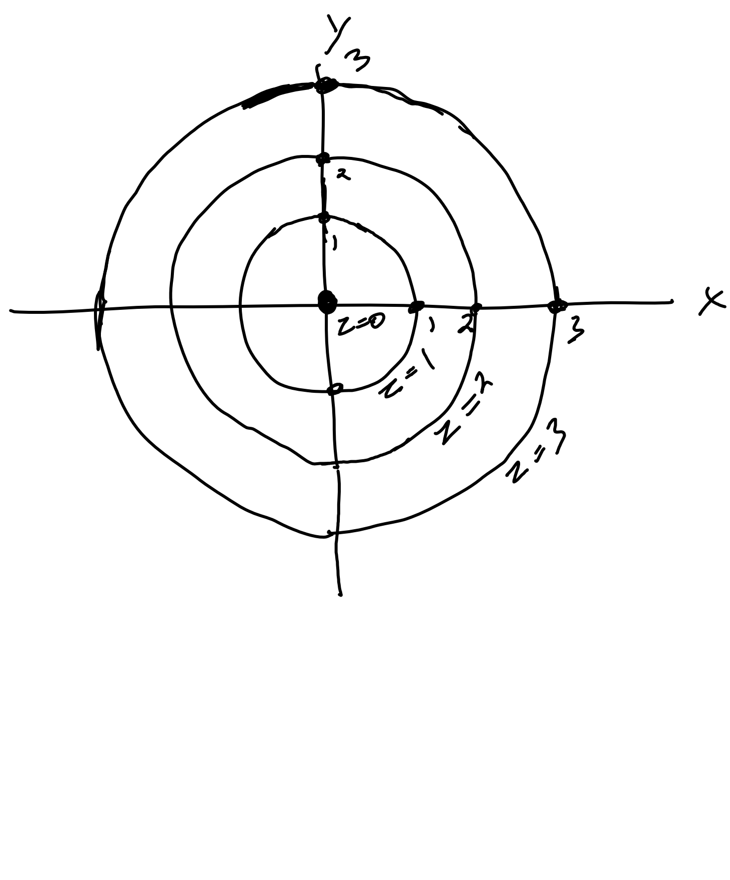
$1 = x^2+y^2$

$z=3$ $\quad 2 = \sqrt{x^2+y^2}$

$4 = x^2+y^2$

$z=3$ $\quad 3 = \sqrt{x^2+y^2}$

$9 = x^2+y^2$



example

Make a contour map for $z = x^2 + y^2$ using values $z = 0, 1, 2, 3$

$z = 0$

$0 = x^2 + y^2$

$z = 1$

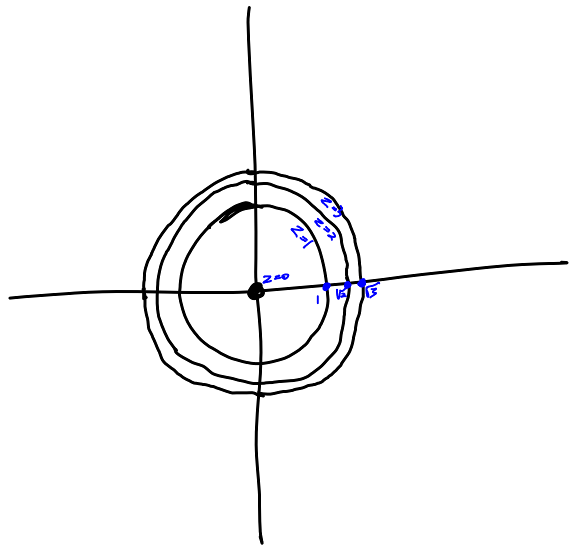
$1 = x^2 + y^2$

$z = 2$

$2 = x^2 + y^2$

$z = 3$

$3 = x^2 + y^2$

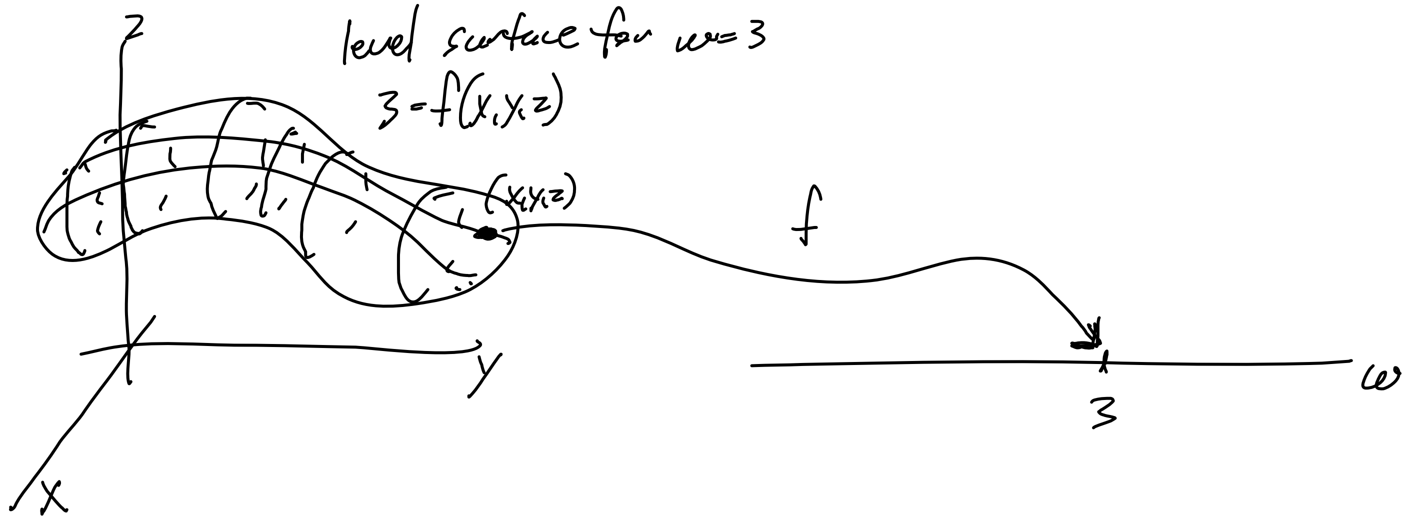


Functions of 3 variables

* Given a function $f(x,y,z)$ its domain is the largest 3-dimensional set of coordinates (x,y,z) for which the function $f(x,y,z)$ is defined.

* The range for $f(x,y,z)$ is again like functions of one and two variables, some subset of the real number line.

* The graph $w = f(x,y,z)$ can be conceptualized with level surfaces. Pick a fixed value for $w = k$. Then $k = f(x,y,z)$ defines a surface in 3D.



* Another way of thinking of a function $f(x,y,z)$ on domain D in 3 dimensions is that $f(x,y,z)$ represents a variable "density" for D at (x,y,z) in $\frac{\text{units mass}}{\text{unit volume}}$.