Section 3.1 Parametric Curves in 2D and 3D

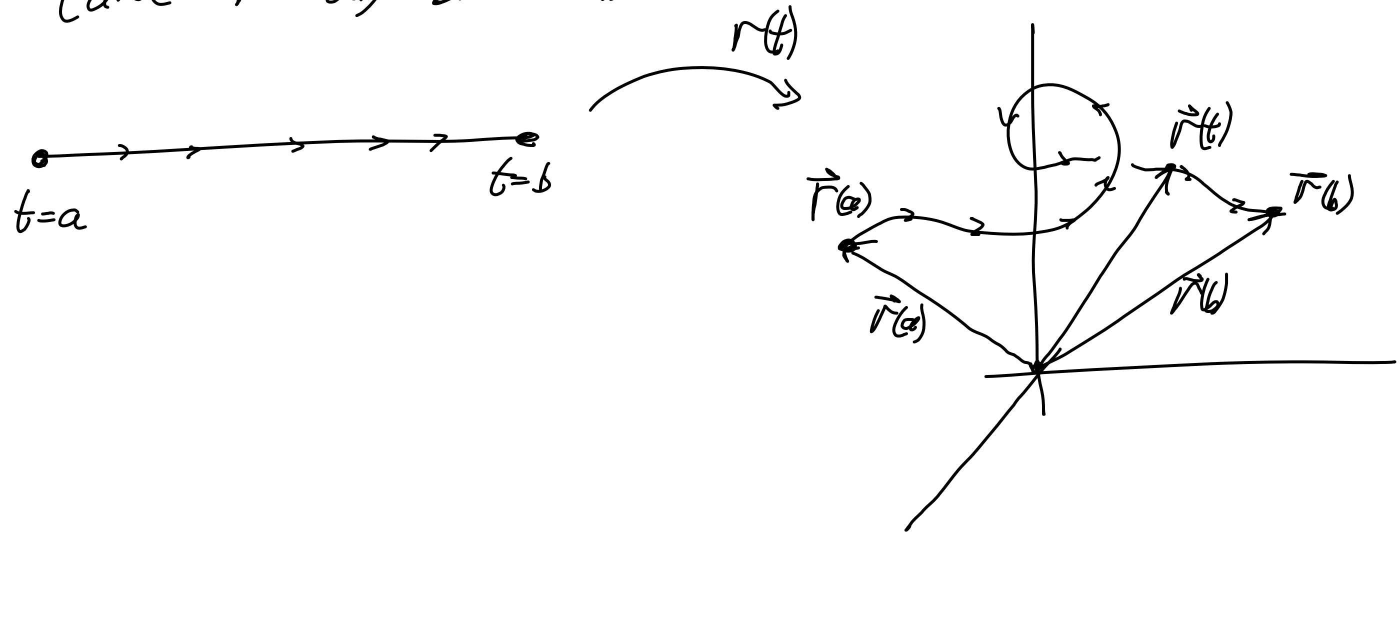
Given a parameter t, a vector function is

$$\vec{r}(t) = \langle x(t), y(t) \rangle \quad \text{in 2D}$$

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle \quad \text{in 3D}$$

Usually we think of t representing time and therefore $\vec{r}(t)$ representing a position of a point at time t.

Graphing all (x,y) or (x,y,z) coordinates for all values of t in some interval produces some 1-dimensional curve in 2D or 3D.



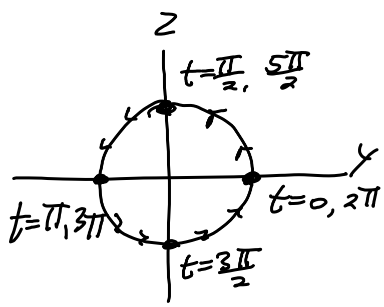
<u>example</u> $\vec{r}(t) = \langle x(t), y(t) \rangle$

$x = \cos(t)$

$y = \sin(t)$

$0 \le t \le 3\pi$

Note $x^2 + y^2 = 1$



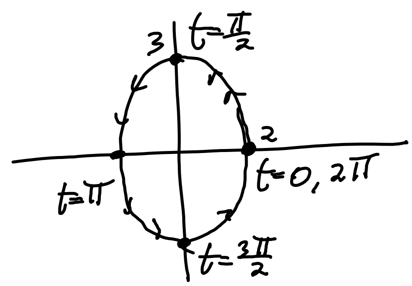
<u>similar example</u>

$x = 2\cos(t)$

$y = 3\sin(t)$

$0 \le t \le 2\pi$

$$\frac{x^2}{4} + \frac{y^2}{9} = \cos^2(t) + \sin^2(t) = 1$$



<u>example</u>

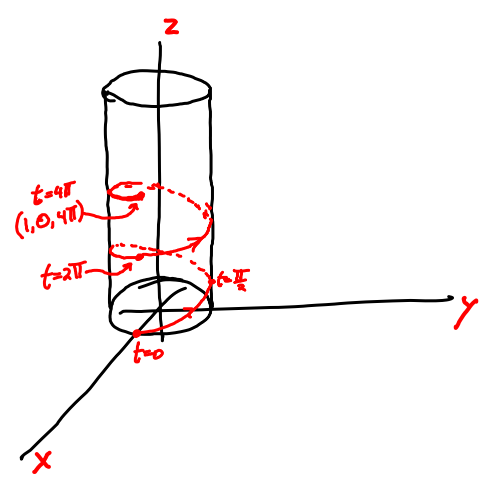
$x = \cos(t)$

$y = \sin(t)$

$z = t$

$0 \le t \le 4\pi$

Because $x^2 + y^2 = 1$
The curve given
by $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$
must lie on the
cylinder $x^2 + y^2 = 1$.



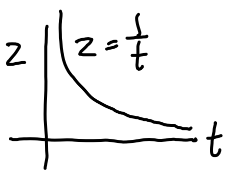
similar example

$x = \cos(t)$

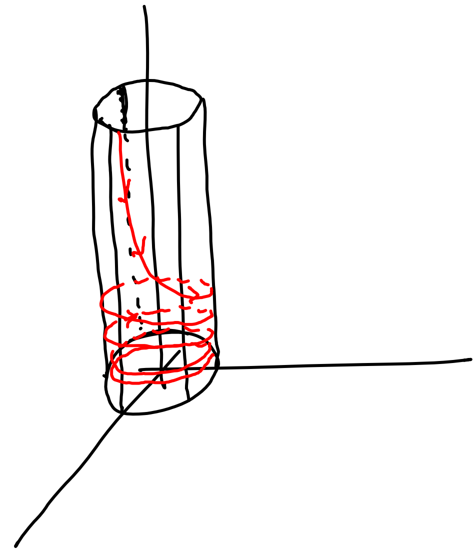
$y = \sin(t)$

$z = \frac{1}{t}$

$t > 0$



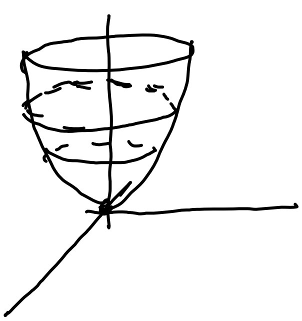
Because $x^2+y^2=1$
The curve given by $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ must live on the cylinder $x^2+y^2=1$.



example

$$\boxed{\begin{aligned} x &= t \\ y &= t \\ z &= 2t^2 \end{aligned}} \quad t \geq 0$$

$z = x^2+y^2$



Because $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ satisfies
$x^2+y^2=z^2$
The curve given by $\vec{r}(t)$ lives on this surface.