Discussion problems for next time

Sections 4.6, 4.7 from the syllabus.

Exam 2 covers sections 4.1, 4.3-4.7

Exam is scheduled for Monday 10/17

Revised Grading Policy

Lowest Mid-term exam	10%
3 higher mid-term exams	22.5% each
Final exam	22.5%

Grading Scale unchanged

90-100 A

75-90 B

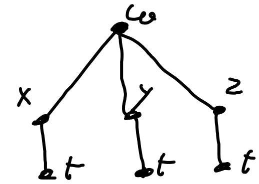
60-75 C

45-60 D

0-45 F

Section 4.5

215) $w = xy\cos(z)$, $x = t$, $y = t^2$, $z = \text{Arcsin}(t)$



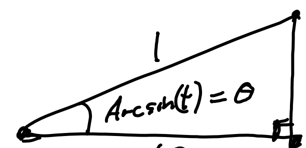
Find $\dfrac{dw}{dt}$,

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$\boxed{\frac{dw}{dt} = y\cos(z) + 2t\,x\cos(z) - \frac{xy\sin(z)}{\sqrt{1-t^2}}}$$

extra

$\cos(\text{Arcsin}(t)) =$



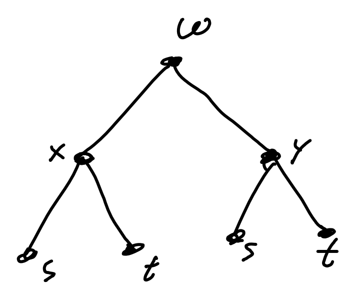
by Pythagorean Theorem

$\cos(\text{Arcsin}(t)) = \sqrt{1-t^2}$

So, actully $w = xy\cos(z)$ in terms of t

is just $w = t^3\sqrt{1-t^2}$

217) $w = 5x^2 + 2y^2$, $x = -3s + t$, $y = s - 4t$



Find 1st partials $\dfrac{\partial w}{\partial s}$, $\dfrac{\partial w}{\partial t}$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$$

$$= 10x(-3) + 4y(1)$$

$$\boxed{= -30x + 4y}$$

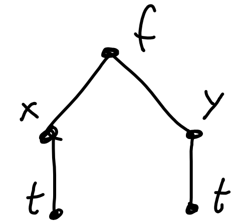
$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$$

$$= 10x(1) + 4y(-4)$$

$$= \boxed{10x - 16y}$$

(220) on your own. Answer $\frac{\partial f}{\partial \theta} = r\cos(\theta) - r\sin(\theta)$

(222) $f = \sqrt{x^2+y^2}\ ,\quad x = t,\ \ y = t^2$



find $\frac{df}{dx}$ using chain Rule and direct substitution.

Confirm that the answers match.

Chain Rule

$$\frac{df}{dx} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= \frac{x}{\sqrt{x^2+y^2}}(1) + \frac{y}{\sqrt{x^2+y^2}}(2t) = \frac{t}{\sqrt{t^2+t^4}} + \frac{2t^3}{\sqrt{t^2+t^4}} = \boxed{\frac{2t^3+t}{\sqrt{t^2+t^4}}}$$

direct substitution

$$f = \sqrt{x^2+y^2} = \sqrt{t^2+t^4}$$

$$\frac{df}{dt} = \cancel{\tfrac{1}{2}}\left(t^2+t^4\right)^{-\frac{1}{2}}\left(\cancel{2}t+\cancel{4}^{\,2}t^3\right) = \boxed{\frac{t+2t^3}{\sqrt{t^2+t^4}}}$$

Same

(225) on your own. In this problem $\frac{df}{dt} = 1$.

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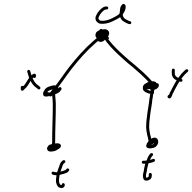
$u = e^x \sin(y)$, $x = -\ln(2t)$, $y = \pi t$



Remember

$\frac{d}{dt}\ln(g(t)) = \frac{g'(t)}{g(t)}$

Find $\left.\frac{du}{dt}\right|_{(x,y)=\left(\ln(2),\, \frac{\pi}{4}\right)}$

Before proceeding notice that $\pi t = \frac{\pi}{4}$ when $t = \frac{1}{4}$

also that $-\ln\left(2\frac{1}{4}\right) = -\ln\left(\frac{1}{2}\right) = \ln(2)$

Thus $(x,y) = \left(\ln(2), \frac{\pi}{4}\right)$ corresponds to $t = \frac{1}{4}$.

So $$\left.\frac{du}{dt}\right|_{(x,y)=\left(\ln(2),\, \frac{\pi}{4}\right)} = \left.\frac{du}{dt}\right|_{t=\frac{1}{4}}$$

Now $$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$= e^x \sin(y)\left(\frac{-1}{t}\right) + e^x \cos(y)\,\pi$$

$$\left.\frac{du}{dt}\right|_{\left(\ln(2),\, \frac{\pi}{4}\right)} = \cancel{2}\frac{\sqrt{2}}{\cancel{2}}(-4) + \cancel{2}\frac{\sqrt{2}}{\cancel{2}}\,\pi =$$

$$\boxed{= \sqrt{2}(\pi - 4)}$$

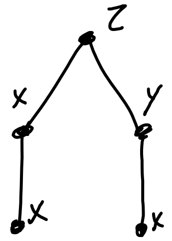
Given a mixed equation in x and y

$f(x,y) = 0$, y is implicitly a function of x.

Also we can think of this as the level curve for $f(x,y) = z$

when $z = 0$.

now



0 because $z=0$

$$\frac{\partial \cancel{z}}{\partial x} = \frac{\partial z}{\partial x}\underbrace{\frac{d\cancel{x}}{d\cancel{x}}}_{=1} + \frac{\partial z}{\partial y}\frac{dy}{dx}$$

$$0 = f_x + f_y \frac{dy}{dx}$$

$$\boxed{-\frac{f_x}{f_y} = \frac{dy}{dx}}$$

So Given $\sin(x+y) + \cos(x-y) = 4$

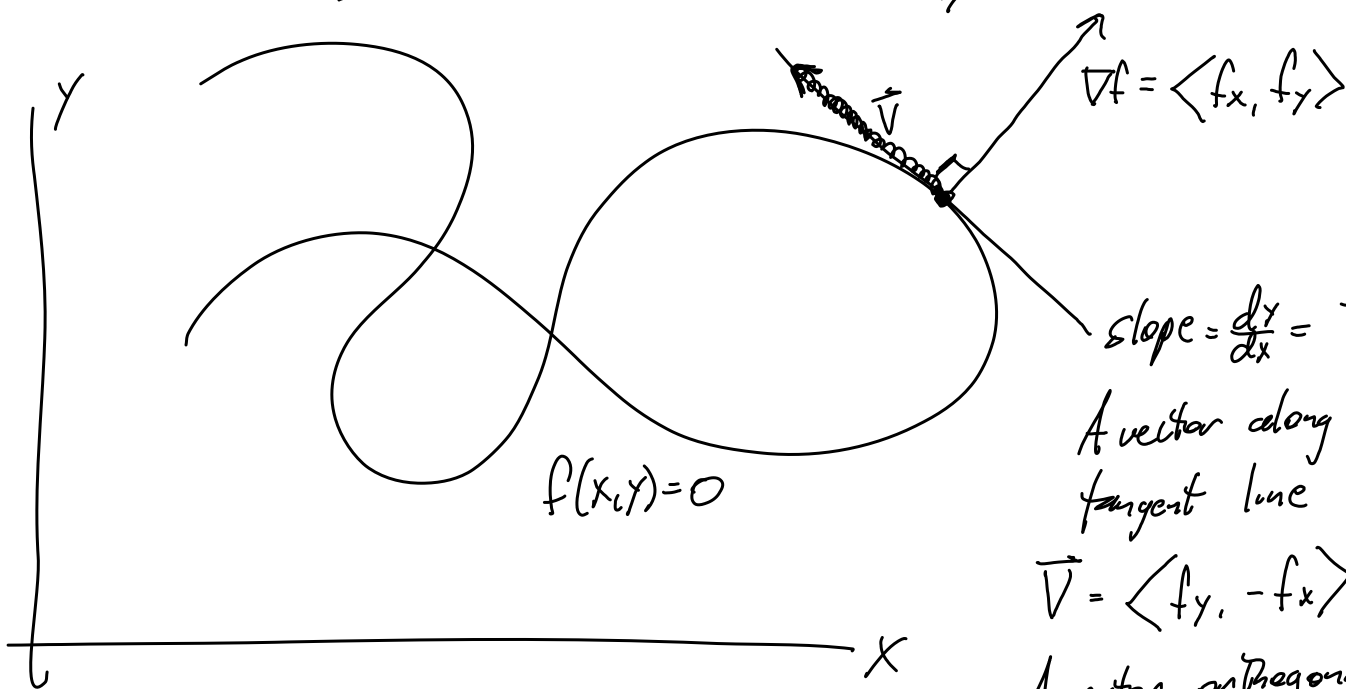
use $f(x,y) = \sin(x+y) + \cos(x-y) - 4 = 0$

$$\boxed{\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{\cos(x+y) - \sin(x-y)}{\cos(x+y) + \sin(x-y)}}$$

234 $\underbrace{xe^y + ye^x - 2x^2y}_{f(x,y)} = 0$ find $\frac{dy}{dx}$ using partial derivatives.

$$\frac{dy}{dx} = -\frac{f_x}{f_y} = \boxed{-\frac{e^y + ye^x - 4xy}{xe^y + e^x - 2x^2}}$$

Another interesting fact about $\frac{dy}{dx} = \frac{-f_x}{f_y}$



slope $= \frac{dy}{dx} = \frac{-f_x}{f_y}$

A vector along this tangent line is

$\vec{V} = \langle f_y, -f_x \rangle$

A vector orthogonal to $\vec{V}$ is

$$\boxed{\nabla f = \langle f_x, f_y \rangle}$$

(245) On your own.

(250) $f(x,y) = x^2y - 2y^3$ now consider

$$\boxed{f(tx,ty)} = (tx)^2(ty) - 2(ty)^3 = t^3x^2y - 2t^3y^3 = t^3(x^2y - 2y^3) = \boxed{t^3 f(x,y)}$$

So $f(x,y)$ is called <u>homogeneous</u> of degree 3.

This should satisfy $x f_x + y f_y = 3f$. Let's check.

$f_x = 2xy$

$f_y = x^2 - 6y^2$

$$\boxed{x f_x + y f_y} = 2x^2y + x^2y - 6y^3 = 3x^2y - 6y^3 = 3(x^2 - 2y^3) = \boxed{3f}$$

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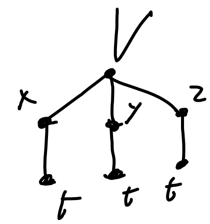
$$V = \frac{\pi}{3} z\left(x^2 + xy + y^2\right) \quad \text{where}$$



Find $\left.\dfrac{dV}{dt}\right|_{(x,y,z)=(10,12,18)}$ where $\dfrac{dx}{dt} = 1$ in/min $\quad \dfrac{dy}{dt} = 1$ in/min $\quad \dfrac{dz}{dt} = -5$ in/min

all in inches.

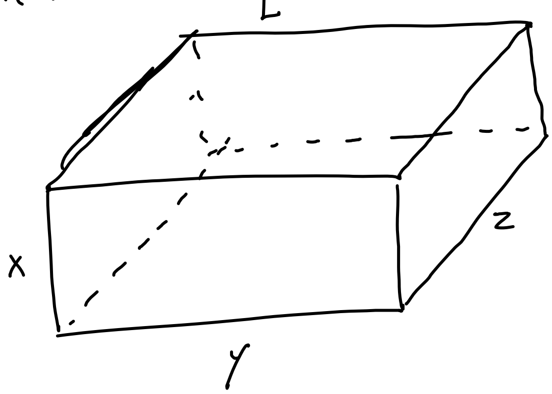
$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt} + \frac{\partial V}{\partial z}\frac{dz}{dt}$$

$$\boxed{\frac{dV}{dt} = \frac{\pi}{3} z(2x+y)\frac{dx}{dt} + \frac{\pi}{3} z(2y+x)\frac{dy}{dt} + \frac{\pi}{3}\left(x^2+xy+y^2\right)\frac{dz}{dt}}$$

Now

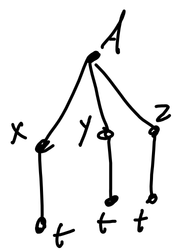
$$\left.\frac{dV}{dt}\right|_{(10,12,18)} = \frac{\pi}{3}\Big[576(1) + 612(1) + 364(-5)\Big] = -661.8 \ \frac{\text{inches}^3}{\text{min}}$$

255



Surface Area $A = 2xy + 2yz + 2xz$

$$\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = \frac{1}{2} \text{ in/min}$$



Find $\left.\dfrac{dA}{dt}\right|_{(2,3,1)}$

$$\frac{dA}{dt} = \frac{\partial A}{\partial x}\frac{dx}{dt} + \frac{\partial A}{\partial y}\frac{dy}{dt} + \frac{\partial A}{\partial z}\frac{dz}{dt}$$

$$\boxed{\frac{dA}{dt} = (2y+2z)\frac{dx}{dt} + (2x+2z)\frac{dy}{dt} + (2x+2y)\frac{dz}{dt}}$$

Now $\left.\frac{dA}{dt}\right|_{(2,3,1)} = 8\frac{1}{2} + 6\frac{1}{2} + 10\frac{1}{2} = \boxed{12\ \frac{\text{inches}^2}{\text{min}}}$

257 omit

try 253 on your own for next time.

We will definitely have a related-rates problem on exam 2.