Discussion problems for next time

Sections 4.5, 4.6 from the syllabus.

Exam 2 covers sections 4.1, 4.3-4.7

Exam is scheduled for Monday 10/17

Revised Grading Policy

Lowest Mid-term exam	10%
3 higher mid-term exams	22.5% each
Final exam	22.5%

Grading Scale unchanged

90-100 A

75-90 B

60-75 C

45-60 D

0-45 F

4.4

$\boxed{167, 169, 172 \quad \text{Omit}}$

(174) $z = e^{7x^2+4y^2}$ find equation of tangent plane and parametric equations of normal line at point $(x, y, z) = (0, 0, 1)$

$$\frac{\partial z}{\partial x} = e^{7x^2+4y^2}(14x) = 14xe^{7x^2+4y^2}$$

$$\frac{\partial z}{\partial y} = 8ye^{7x^2+4y^2}$$

$$\left.\frac{\partial z}{\partial x}\right|_{(0,0,1)} = 0$$

$$\left.\frac{\partial z}{\partial y}\right|_{(0,0,1)} = 0$$

Tangent plane

$$z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$z - 1 = 0$$

$$z = 1$$

Normal line

$$\vec{n} = \langle -f_x(a,b), -f_y(a,b), 1 \rangle = \langle 0, 0, 1 \rangle$$

$x = 0 + 0t$
$y = 0 + 0t$
$z = 1 + t$

$$\boxed{\begin{aligned} x &= 0 \\ y &= 0 \\ z &= 1+t \end{aligned}}$$

(176) $\boxed{x^2+4y^2=z^2}$ find tangent plane and normal line at

$(x,y,z)=(3,2,5)$

must write as $z=f(x,y)$

$$z=\sqrt{x^2+4y^2}$$

$$\frac{\partial z}{\partial x}=\frac{1}{\cancel{2}}(x^2+4y^2)^{-\frac{1}{2}}(\cancel{2}x)=\frac{x}{\sqrt{x^2+4y^2}} \qquad \left.\frac{\partial z}{\partial x}\right|_{(x,y)=(3,2)}=\frac{3}{5}$$

$$\frac{\partial z}{\partial y}=\frac{1}{\cancel{2}}(x^2+4y^2)^{-\frac{1}{2}}(\cancel{8}^{4}y)=\frac{4y}{\sqrt{x^2+4y^2}} \qquad \left.\frac{\partial z}{\partial y}\right|_{(x,y)=(3,2)}=\frac{8}{5}$$

Tangent Plane

$$z-f(a,b)=f_x(a,b)(x-a)+f_y(a,b)(y-b)$$

$$\boxed{z-5=\frac{3}{5}(x-3)+\frac{8}{5}(y-2)}$$

Normal line

$$\vec{n}=\langle -f_x,-f_y,1\rangle=\left\langle -\frac{3}{5},-\frac{8}{5},1\right\rangle$$

$$\boxed{\begin{aligned} x&=3-\tfrac{3}{5}t\\ y&=2-\tfrac{8}{5}t\\ z&=5+t\end{aligned}}$$

(183) $z = 5x^2 - 2y^2$ find the equations of normal line at $(2,1,18)$

$$\frac{\partial z}{\partial x} = 10x \qquad \left.\frac{\partial z}{\partial x}\right|_{(x,y)=(2,1)} = 20$$

$$\frac{\partial z}{\partial y} = -4y \qquad \left.\frac{\partial z}{\partial x}\right|_{(x,y)=(2,1)} = -4$$

$$\vec{n} = \langle -f_x(2,1), -f_y(2,1), 1 \rangle$$

$$\vec{n} = \langle -20, 4, 1 \rangle$$

$$\boxed{\begin{aligned} x &= 2 - 20t \\ y &= 1 + 4t \\ z &= 18 + t \end{aligned}}$$

(184) $x^2 - 8xyz + y^2 + 6z^2 = 0$ at $(1,1,1)$ find normal line.

normal vector $\vec{n} = \left\langle -\left.\frac{\partial z}{\partial x}\right|_{(1,1,1)}, -\left.\frac{\partial z}{\partial y}\right|_{(1,1,1)}, 1 \right\rangle$

but the surface isn't written as $z = f(x,y)$ explicitly.

However a mixed equation for x, y, z may be considered as a function $z = f(x,y)$ implicity.

So we apply $\frac{\partial}{\partial x}$ to each side of the equation to find $\frac{\partial z}{\partial x}$, and apply $\frac{\partial}{\partial y}$ to find $\frac{\partial z}{\partial y}$.

$\boxed{\text{Find } \frac{\partial z}{\partial x}}$

$$\frac{\partial}{\partial x}\left(x^2 - 8xyz + y^2 + 6z^2\right) = \frac{\partial}{\partial x}0$$

$$\frac{\partial}{\partial x}x^2 - \frac{\partial}{\partial x}8xyz + \frac{\partial}{\partial x}y^2 + \frac{\partial}{\partial x}6z^2 = 0$$

$$2x - 8yz - 8xy\frac{\partial z}{\partial x} + 0 + 12z\frac{\partial z}{\partial x} = 0$$

$$2x - 8yz = 8xy\frac{\partial z}{\partial x} - 12z\frac{\partial z}{\partial x}$$

$$\boxed{\frac{2x - 8yz}{8xy - 12z} = \frac{\partial z}{\partial x}}$$

$$\left.\frac{\partial z}{\partial x}\right|_{(1,1)} = \frac{2-8}{8-12} = \frac{6}{4} = \frac{3}{2}$$

$\boxed{\text{Find } \frac{\partial z}{\partial y}}$

$$\frac{\partial}{\partial y}\left(x^2 - 8xyz + y^2 + 6z^2\right) = \frac{\partial}{\partial y}0$$

$$\frac{\partial}{\partial y}x^2 - \frac{\partial}{\partial y}8xyz + \frac{\partial}{\partial y}y^2 + \frac{\partial}{\partial y}6z^2 = 0$$

$$0 - 8xz - 8xy\frac{\partial z}{\partial y} + 2y + 12z\frac{\partial z}{\partial y} = 0$$

$$2y - 8xz = 8xy\frac{\partial z}{\partial y} - 12z\frac{\partial z}{\partial y}$$

$$\boxed{\frac{2y - 8xz}{8xy - 12z} = \frac{\partial z}{\partial y}}$$

$$\left.\frac{\partial z}{\partial y}\right|_{(1,1)} = \frac{-6}{-4} = \frac{3}{2}$$

So $\vec{n} = \left\langle -\left.\frac{\partial z}{\partial x}\right|_{(1,1)}, -\left.\frac{\partial z}{\partial y}\right|_{(1,1)}, 1 \right\rangle$

$$\vec{n} = \left\langle -\frac{3}{2}, -\frac{3}{2}, 1 \right\rangle$$

$$\boxed{\begin{aligned} x &= 1-\tfrac{3}{2}t \\ y &= 1-\tfrac{3}{2}t \\ z &= 1+t \end{aligned}}$$

Extra Equation of tangent plane

$$\boxed{z-1 = \tfrac{3}{2}(x-1)+\tfrac{3}{2}(y-1)}$$

(186) almost exactly the same as 174

186 $\quad z = e^{4x^2+6y^2}$ at $(0,0,1)$

174 $\quad z = e^{2x^2+4y^2}$ at $(0,0,1)$

(194) $\quad z = \dfrac{xy}{x+y}$ differential $\quad \Delta z = \left.\dfrac{\partial z}{\partial x}\right|_{(a,b)}\Delta x + \left.\dfrac{\partial z}{\partial y}\right|_{(a,b)}\Delta y$

where the base point of tangency is $(x,y,z) = \left(10, 15, \frac{150}{25}\right) = (10,15,6)$

$$\frac{\partial z}{\partial x} = \frac{y(x+y)-(1)xy}{(x+y)^2} = \frac{y^2}{(x+y)^2} \qquad \left.\frac{\partial z}{\partial x}\right|_{(10,15)} = \frac{225}{625} = \frac{36}{100} = \frac{9}{25}$$

$$\frac{\partial z}{\partial y} = \text{------} = \frac{x^2}{(x+y)^2} \qquad \left.\frac{\partial z}{\partial y}\right|_{(10,15)} = \frac{100}{625} = \frac{4}{25}$$

because $z = \frac{xy}{x+y}$ is unchanged when switching x and y.

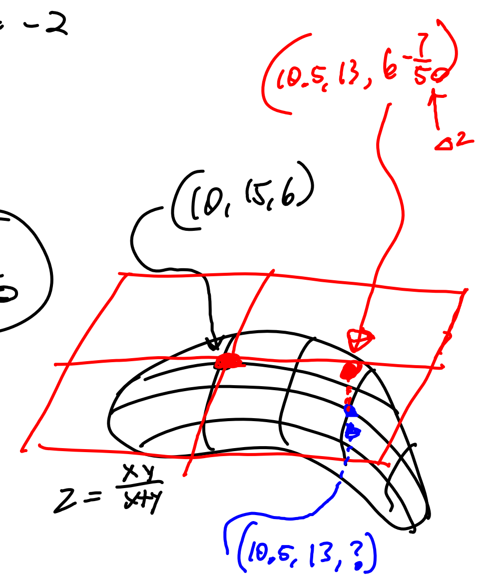
x goes from 10 to 10.5 so $\Delta x = .5$

y goes from 15 to 13 so $\Delta y = -2$

Differential

$$\Delta z = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$$

$$\Delta z = \frac{9}{25}(.5) + \frac{4}{25}(-2) = \frac{-3.5}{25} = \frac{-7}{50}$$



(194) $z=\sqrt{4-x^2-y^2}$ approximate the change in z for $z=\sqrt{4-x^2-y^2}$ using the differential Δz as $(x\ y)$ moves from $(1,1) \longrightarrow (1.01, 0.97)$

$\Delta x = .01$

$\Delta y = -.03$

$$\frac{\partial z}{\partial x} = \cancel{\tfrac{1}{2}}\left(4-x^2-y^2\right)^{-\frac{1}{2}}\left(-\cancel{2}x\right) = \frac{-x}{\sqrt{4-x^2-y^2}}$$

$$\frac{\partial z}{\partial x}\Big|_{(1,1)} = \frac{-1}{\sqrt{4-1-1}} = -\frac{1}{\sqrt{2}}$$

$$\frac{\partial z}{\partial y} = \dots\dots = \frac{-y}{\sqrt{4-x^2-y^2}}$$

$$\frac{\partial z}{\partial y}\Big|_{(1,1)} = -\frac{1}{\sqrt{2}}$$

$$\Delta z = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$$

$$\underbrace{z - f(a,b)}_{\Delta z} = f_x(a,b)\underbrace{(x-a)}_{\Delta x} + f_y(a,b)\underbrace{(y-b)}_{\Delta y}$$

So $\Delta z = \frac{-1}{\sqrt{2}}\Delta x - \frac{1}{\sqrt{2}}\Delta y$

$$\Delta z = \frac{-1}{\sqrt{2}}(.01) + (-.03)\frac{-1}{\sqrt{2}}$$

$$\boxed{\approx .0141421}$$

So on the tangent plane, the old z-coordinate is $z(1,1) = \sqrt{2}$

, the new z-coordinate is

$\sqrt{2} + \Delta z \approx$

$\sqrt{2} + .0141421 =$

$1.4283557.$

The actual value of the function is pretty close

$z(1.01, 0.97) \approx 1.42794\dots$

209 on your own , 211 omit.