Discussion problems for next time

Section 4.7 from the syllabus.

Exam 2 covers sections 4.1, 4.3-4.7

Exam is scheduled for Monday 10/17

Section 4.6

262 $f(x,y)=y^2\sin(2x)$ find $D_{\vec{u}}\left(f\left(\frac{\pi}{4},2\right)\right)$ where is is in the direction of $\langle 5,12\rangle$.

Since $\vec{u}$ must be a unit vector.

$$\vec{u}=\frac{1}{13}\langle 5,12\rangle=\left\langle \frac{5}{13},\frac{12}{13}\right\rangle$$

$f_x = 2y^2\cos(2x)$

$f_y = 2y\sin(2x)$

So $\nabla f\left(\frac{\pi}{4},2\right)=\langle 0,4\rangle$

$f_x\left(\frac{\pi}{4},2\right)=0$

$f_y\left(\frac{\pi}{4},2\right)=4$

So $D_{\vec{u}}\left(f\left(\frac{\pi}{4},2\right)\right)=\nabla f\cdot\vec{u}$

$$=\langle 0,4\rangle\cdot\left\langle\frac{5}{13},\frac{12}{13}\right\rangle=\frac{48}{13}$$

265 $h(x,y,z)=xyz$

$h_x=yz \quad h_y=xz \quad h_z=xy$

$h_x(2\,1\,1)=1$

$h_y(2\,1\,1)=2$

$h_z(2\,1\,1)=2$

So $\nabla h(2\,1\,1)=\langle h_x,h_y,h_z\rangle=\langle 1,2,2\rangle$

Calculate $D_{\vec{u}}(h(2,1,1))$ where $\vec{u}$ is in the direction of $\langle 2,1,-1\rangle$

Since $\vec{u}$ is a unit vector $\vec{u} = \frac{1}{\sqrt{6}}\langle 2,1,-1\rangle = \left\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right\rangle$.

So $D_{\vec{u}}(h(2,1,1)) = \nabla h(2,1,1)\cdot\vec{u} = \langle 1,2,2\rangle\cdot\left\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right\rangle = \frac{2}{\sqrt{6}}$

(272) $f(x,y) = x^2y$

$f_x = 2xy$

$f_y = x^2$

$f_x(-5,5) = -50$

$f_y(-5,5) = 25$

$\nabla f(-5,5) = \langle -50, 25\rangle$

$\vec{u}$ is the unit vector in the direction of $\vec{v} = \langle 3,-4\rangle$.

So $\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \left\langle \frac{3}{5}, \frac{-4}{5}\right\rangle$.

$D_{\vec{u}}(f(-5,5)) = \nabla f\cdot\vec{u}$

$= \langle -50, 25\rangle\cdot\left\langle \frac{3}{5}, -\frac{4}{5}\right\rangle$

$= -30-20 = -50$

(275, 278) on your own

(284) $f(x,y) = x^2+3y^2$

$f_x = 2x$

$f_y = 6y$

$\nabla f(1,1) = \langle 2,6\rangle$

$\vec{v} = \overrightarrow{PQ} = \langle 3,4\rangle$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \left\langle \frac{3}{5}, \frac{4}{5}\right\rangle$

$D_{\vec{u}}(f(1,1)) = \nabla f\cdot\vec{u}$

$= \langle 2,6\rangle\cdot\left\langle \frac{3}{5}, \frac{4}{5}\right\rangle$

$= \frac{30}{5} = 6$

287, 291 on your own.

294 $f(x,y) = x^2 + xy + y^2$

find the greatest directional derivative at point $(-5,-4)$.

$f_x = 2x + y \quad f_x(-5,-4) = -14$

$f_y = x + 2y \quad f_y(-5,-4) = -13$

$\nabla f(-5,-4) = \langle -14, -13 \rangle$

The largest possible value of $D_{\vec{u}}(f(-5,-4)) = |\nabla f| = \sqrt{365}$
≈ 19.10

in the direction of ∇f which is

$\vec{u} = \frac{1}{\sqrt{365}} \nabla f = \left\langle \frac{-14}{\sqrt{365}}, \frac{-13}{\sqrt{365}} \right\rangle$

Extra comment,

The smallest possible value of $D_{\vec{u}}(f(-5,-4)) = -|\nabla f| = -\sqrt{365}$
≈ -19.10

in the direction of $-\nabla f$ which is

$\vec{u} = \frac{-1}{\sqrt{365}} \nabla f = \left\langle \frac{14}{\sqrt{365}}, \frac{13}{\sqrt{365}} \right\rangle$

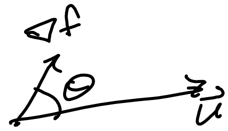
(296) $f(x,y) = \text{Arctan}\left(\frac{y}{x}\right)$ find max value $D_{\vec{u}}(f(-9,9))$

$f_x = \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{-y}{x^2} = \frac{-y}{x^2+y^2}$ $\qquad f_x(-9,9) = \frac{-9}{162} = \frac{-1}{18}$

$f_y = \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{1}{x}$ $\qquad f_y(-9,9) = \frac{-1}{18}$

$\nabla f(-9,9) = \left\langle \frac{-1}{18}, \frac{-1}{18}\right\rangle =$

The largest directional derivative is therefore $|\nabla f| = \frac{1}{\sqrt{162}} = \frac{1}{9\sqrt{2}}$
in the direction of ∇f itself which is $\vec{u} = \frac{1}{\nabla f}\nabla f = 9\sqrt{2}\left\langle\frac{-1}{18},\frac{-1}{18}\right\rangle$
$= \left\langle \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right\rangle$



extra comment

Remember $D_{\vec{u}}(f(a,b)) = \nabla f\cdot\vec{u} = |\nabla f|\,|\vec{u}|\cos(\theta) = |\nabla f|\cos(\theta)$

and max value of $\cos(\theta) = 1$ at $\theta = 0°$.

(300) Again the max-value directional derivative is $|\nabla f|$.

$f(x,y) = \sqrt{x^2+2y}$

$f_x = \frac{x}{\sqrt{x^2+2y}}$ $\qquad f_x(4,10) = \frac{4}{6} = \frac{2}{3}$

$f_y = \frac{1}{\sqrt{x^2+2y}}$ $\qquad f_y(4,10) = \frac{1}{6}$

So $\nabla f(4,10) = \left\langle \frac{2}{3}, \frac{1}{6} \right\rangle$

So max value of $D_{\vec{u}}(f(4,10))$ is $|\nabla f(4,10)| = \sqrt{\frac{4}{9}+\frac{1}{36}}$

$$= \sqrt{\frac{17}{36}}$$

$$= \frac{\sqrt{17}}{6}$$

In the direction of $\boxed{\vec{u} = \frac{6}{\sqrt{17}}\left\langle \frac{2}{3}, \frac{1}{6} \right\rangle = \left\langle \frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}} \right\rangle}$

(302) Find the equation of the tangent plane and normal line to the surface $12 = 4x^2 - 2y^2 + z^2$ at the point $(2,2,2)$.

Remember, when $z = f(x,y)$ explicitly, we calculate the tangent plane as $z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b)$.

The normal vector in this case turns out to be $\vec{n} = \langle -f_x, -f_y, 1 \rangle$.

However for mixed equations $K = f(x,y,z)$ we now know that $\nabla f(a,b,c) = \langle f_x, f_y, f_z \rangle$ is perpendicular to the surface and therefore can be used as a normal vector

In This Case, $12 = \underbrace{4x^2 - 2y^2 + z^2}_{f(x,y,z)}$

$f_x = 8x$

$f_y = -4y$

$f_z = 2z$

$\nabla f(2,2,2) = \langle 16, -8, 4 \rangle = \vec{n}$

Tangent plane

$\vec{n} \cdot \langle x, y, z \rangle = \vec{n} \cdot \langle 2, 2, 2 \rangle$

$\boxed{16x - 8y + 4z = 24}$

normal Line

$$\boxed{\begin{aligned} x &= 2 + 16t \\ y &= 2 - 8t \\ z &= 2 + 4t \end{aligned}}$$

306 omit.