Exam 3 covers sections 5.1–5.6

scheduled for Monday 11/7

Sec 5.5

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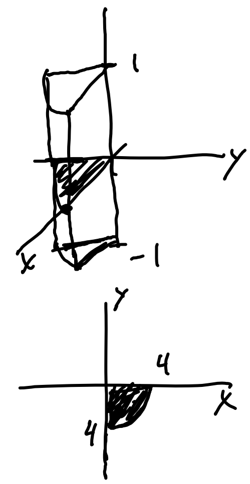
$$\iiint_R xz^2\,dV$$

R is defined by $x^2+y^2\le 16$ cylinder of radius 4



$$\boxed{\begin{array}{l}\frac{3\pi}{2}\le\theta\le 2\pi\\ 0\le r\le 4\\ -1\le z\le 1\end{array}}$$

$$\iiint_R xz^2\,dV = \int_{3\pi/2}^{2\pi}\int_0^4\int_{-1}^{1} xz^2\,dz\,r\,dr\,d\theta = \int_{3\pi/2}^{2\pi}\int_0^4 \frac{1}{3}xz^3\Big]_{z=-1}^{1} r\,dr\,d\theta = \frac{3}{3}\int_{3\pi/2}^{2\pi}\int_0^4 x\,r\,dr\,d\theta$$

$$= \frac{2}{3}\int_{3\pi/2}^{2\pi}\int_0^4 r^2\cos(\theta)\,dr\,d\theta = \frac{2}{3}\int_{3\pi/2}^{2\pi}\frac{1}{3}r^3\cos(\theta)\Big]_{r=0}^{r=4} d\theta = \frac{128}{9}\int_{3\pi/2}^{2\pi}\cos(\theta)\,d\theta$$

$$= \frac{128}{9}\sin(\theta)\Big]_{\theta=3\pi/2}^{2\pi} = \frac{128}{9}\big(0-(-1)\big) = \frac{128}{9}$$

(245)

$$\iiint_R e^{\sqrt{x^2+y^2}}\, dV$$

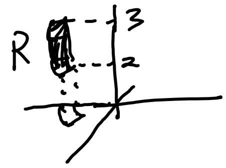
R is defined by

$1 \le x^2+y^2 \le 4$

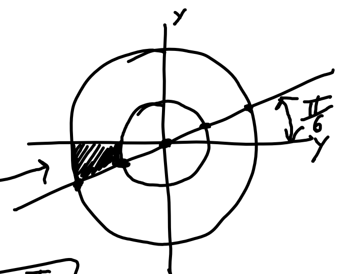
$y \le 0$

$x \le \sqrt{3}\, y$

$2 \le z \le 3$



In just the xy-plane



$x = y\sqrt{3}$ or $\frac{1}{\sqrt{3}} = \frac{y}{x}$

or $\frac{1}{\sqrt{3}} = \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

So for $r=1$

$\cos(\theta) = \frac{\sqrt{3}}{2}$

$\sin(\theta) = \frac{1}{2}$

$$\boxed{\begin{array}{c} \pi \le \theta \le \frac{7\pi}{6} \\ 1 \le r \le 2 \end{array}}$$

$$R = \boxed{\begin{array}{c} \pi \le \theta \le \frac{7\pi}{6} \\ 1 \le r \le 2 \\ 2 \le z \le 3 \end{array}}$$

Remember

$$\int_a^b k\, dt = k(b-a)$$

$$\iiint_R e^{\sqrt{x^2+y^2}}\, dV = \int_{\pi}^{\frac{7\pi}{6}}\int_1^2\int_2^3 e^{\sqrt{x^2+y^2}}\, dz\, r\, dr\, d\theta = \int_{\pi}^{\frac{7\pi}{6}}\int_1^2 e^{\sqrt{x^2+y^2}}\, r\, dr\, d\theta = \int_{\pi}^{\frac{7\pi}{6}}\int_1^2 re^r\, dr\, d\theta$$

$$= \int_{\pi}^{\frac{7\pi}{6}} (r-1)e^r \Big]_{r=1}^{r=2} d\theta = \int_{\pi}^{\frac{7\pi}{6}} e^2\, d\theta$$

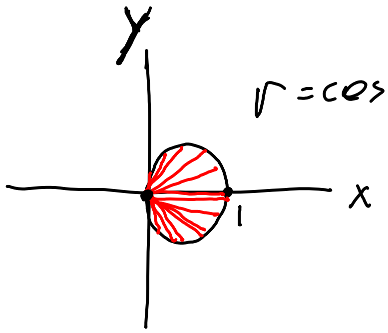
$$\int re^r dr = re^r - \int e^r dr = re^r - e^r = (r-1)e^r$$

$u = r \quad du = dr$

$dv = e^r dr \quad v = e^r$

$$\boxed{= e^2\left(\frac{\pi}{6}\right)}$$

250 E is defined by the cylinder $r=\cos(\theta)$.

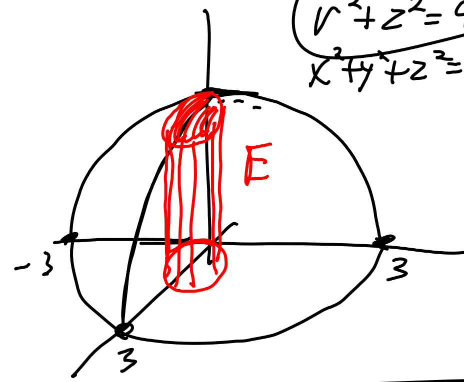


$r=\cos(\theta)$ for $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$

$$\boxed{\begin{array}{l} -\frac{\pi}{2}\le\theta\le\frac{\pi}{2} \\ 0\le r\le\cos(\theta)\end{array}}$$

$r^2+z^2=9$

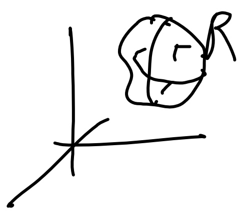
$x^2+y^2+z^2=9$



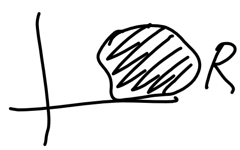
$$E=\boxed{\begin{array}{l} -\frac{\pi}{2}\le\theta\le\frac{\pi}{2} \\ 0\le r\le\cos(\theta) \\ 0\le z\le\sqrt{9-r^2}\end{array}}$$

$$\iiint_E f(x,y,z)\,dV=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\cos(\theta)}\int_0^{\sqrt{9-r^2}} f(r\cos(\theta),r\sin(\theta),z)\,dz\,r\,dr\,d\theta$$

$$\text{Volume}(R)=\iiint_R 1\,dV$$



$$\text{Area}(R)=\iint_R 1\,dA$$



$$\text{length}([a,b])=\int_a^b 1\,dx$$

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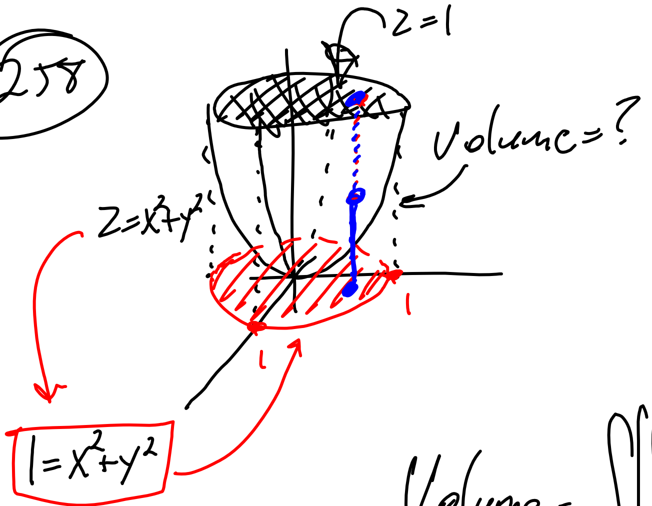

$$R = \boxed{\begin{matrix} 0 \le \theta \le 2\pi \\ 0 \le r \le 1 \\ x^2+y^2 \le z \le 1 \end{matrix}}$$

$$\text{Volume} = \iiint_R 1\, dV = \int_0^{2\pi}\int_0^1\int_{r^2}^1 1\, dz\, r\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 (1-r^2)\, r\, dr\, d\theta = \int_0^{2\pi}\int_0^1 r - r^3\, dr\, d\theta = \int_0^{2\pi} \frac{1}{2}r^2 - \frac{1}{4}r^4 \Big]_{r=0}^{r=1} d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} - \frac{1}{4}\, d\theta = \frac{1}{4}\int_0^{2\pi} d\theta = \frac{2\pi}{4} = \frac{\pi}{2}$$

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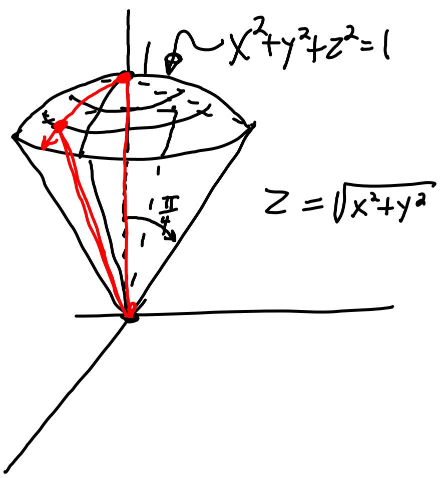
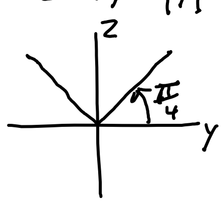


$x=0$

$z=\sqrt{y^2}=|y|$



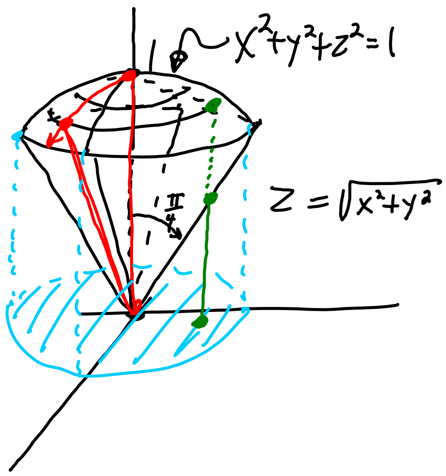
$$R = \boxed{\begin{matrix} 0 \le \varphi \le \frac{\pi}{4} \\ 0 \le \theta \le 2\pi \\ 0 \le \rho \le 1 \end{matrix}}$$

$$\text{Volume} = \iiint_R 1\, dV = \int_0^{\frac{\pi}{4}}\int_0^{2\pi}\int_0^1 1\, \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi = \int_0^{\frac{\pi}{4}}\int_0^{2\pi} \frac{1}{3}\rho^3 \sin(\varphi)\Big]_{\rho=0}^{\rho=1} d\theta\, d\varphi$$

$$= \frac{1}{3}\int_0^{\frac{\pi}{4}}\int_0^{2\pi} \sin(\varphi)\, d\theta\, d\varphi = \frac{2\pi}{3}\int_0^{\frac{\pi}{4}} \sin(\varphi)\, d\varphi = \frac{2\pi}{3}\Big(-\cos(\varphi)\Big]_{\varphi=0}^{\varphi=\frac{\pi}{4}}$$

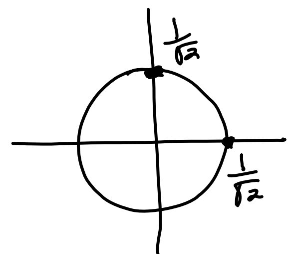
$$= \frac{2\pi}{3}\left(\frac{-\sqrt{2}}{2}+1\right)$$

$$= \boxed{\frac{\pi\left(2-\sqrt{2}\right)}{3}}$$



To describe in cylindrical coordinates, first look at the shadow in xy-plane

$$\sqrt{1-x^2-y^2} = z = \sqrt{x^2+y^2}$$



$$1-x^2-y^2 = x^2+y^2$$

$$\frac{1}{2} = x^2+y^2$$

$$\boxed{\begin{array}{c} 0 \le \theta \le 2\pi \\ 0 \le r \le \frac{1}{\sqrt{2}} \\ \sqrt{x^2+y^2} \le z \le \sqrt{1-x^2-y^2} \end{array}} =$$

$$\boxed{\begin{array}{c} 0 \le \theta \le 2\pi \\ 0 \le r \le \frac{1}{\sqrt{2}} \\ r \le z \le \sqrt{1-r^2} \end{array}} =$$

much harder to use

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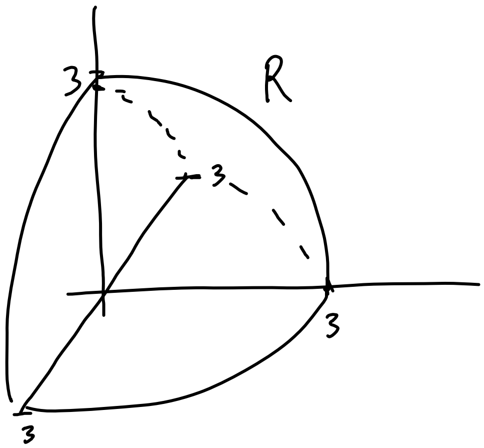

$$R = \boxed{\begin{array}{c} 0 \le \varphi \le \frac{\pi}{2} \\ 0 \le \theta \le \pi \\ 0 \le \rho \le 3 \end{array}}$$

$x = \rho\cos(\theta)\sin(\varphi)$

$y = \rho\sin(\theta)\sin(\varphi)$

$z = \rho\cos(\varphi)$

$x^2+y^2+y^2 = \rho^2$

$$\iiint_R 1-\sqrt{x^2+y^2+z^2}\,dV = \int_0^{\frac{\pi}{2}}\int_0^{\pi}\int_0^{3} (1-\rho)\rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi = \int_0^{\frac{\pi}{2}}\int_0^{\pi}\int_0^{3} (\rho^2-\rho^3)\sin(\varphi)\,d\rho\,d\theta\,d\varphi$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{\pi} \left(\tfrac{1}{3}\rho^3-\tfrac{1}{4}\rho^4\right)\sin(\varphi)\Big]_{\rho=0}^{\rho=3} d\theta\,d\varphi = \int_0^{\frac{\pi}{2}}\int_0^{\pi} \left(9-\tfrac{81}{4}\right)\sin(\varphi)\,d\theta\,d\varphi = \frac{-45}{4}\pi\int_0^{\frac{\pi}{2}}\sin(\varphi)\,d\varphi$$

$$= \frac{45\pi}{4}\cos(\varphi)\Big]_0^{\frac{\pi}{2}} = \frac{-45\pi}{4}$$

The book has a 2 here

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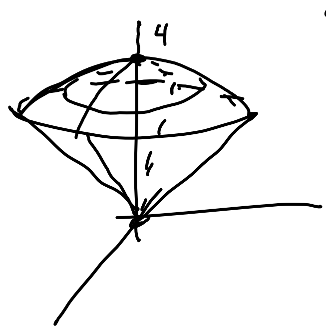
Calculate $\iiint_R z\,dV$ where R is inside Sphere of radius 4

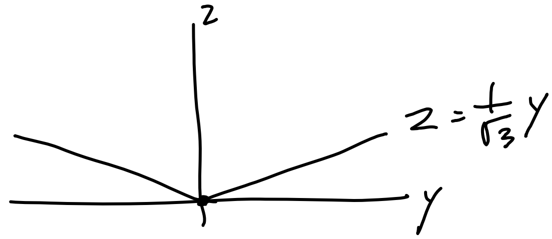
and above the cone $3z^2 = x^2+y^2$

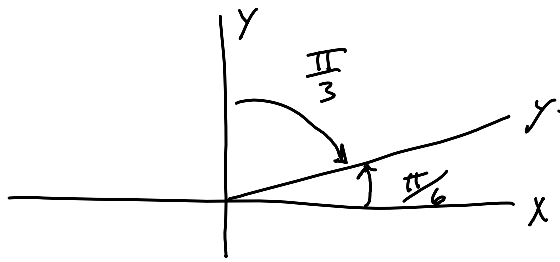
$$z = \frac{1}{\sqrt{3}}\sqrt{x^2+y^2}$$



what is the angle for the cone?

let $x=0$ $\quad z = \frac{1}{\sqrt{3}}|y|$





$\rightarrow \frac{y}{x} = \frac{1}{\sqrt{3}}$

$\sin(\alpha) = \frac{1}{2}$

$\cos(\alpha) = \frac{\sqrt{3}}{2}$

So $\quad R = \boxed{\begin{array}{l} 0 \le \varphi \le \frac{\pi}{3} \\ 0 \le \theta \le 2\pi \\ 0 \le \rho \le 4 \end{array}}$

So $$\iiint_R z\, dV = \int_0^{\frac{\pi}{3}}\int_0^{2\pi}\int_0^{4} \rho\cos(\varphi)\,\rho^2\sin(\varphi)\, d\rho\, d\theta\, d\varphi = \int_0^{\frac{\pi}{3}}\int_0^{2\pi}\int_0^{4} \rho^3\cos(\varphi)\sin(\varphi)\, d\rho\, d\theta\, d\varphi$$

$$= \int_0^{\frac{\pi}{3}}\int_0^{2\pi} \frac{1}{4}\rho^4\cos(\varphi)\sin(\varphi)\Big]_{\rho=0}^{\rho=4} d\theta\, d\varphi = 64\int_0^{\frac{\pi}{3}}\int_0^{2\pi}\cos(\varphi)\sin(\varphi)\, d\theta\, d\varphi =$$

$$= 128\pi\int_0^{\frac{\pi}{3}}\cos(\varphi)\sin(\varphi)\, d\varphi = 128\pi\int_{\varphi=0}^{\varphi=\frac{\pi}{3}} u\, du = 64\pi u^2\Big]_{\varphi=0}^{\varphi=\frac{\pi}{3}}$$

let $u = \sin(\varphi)$

$du = \cos(\varphi)\, d\varphi$

$$= 64\pi \sin^2(\varphi)\Big]_{\varphi=0}^{\varphi=\frac{\pi}{3}}$$

$$= 64\pi\left(\frac{3}{4} - 0\right) = 48\pi$$

286 omit

292 omit.