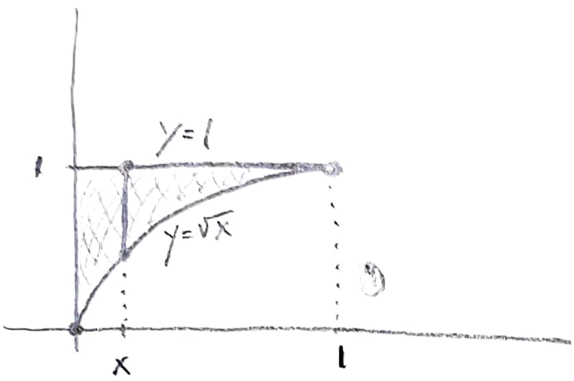
Math2320 – Fall 2022 – Exam 3 Name:____________________

17 pts (1) Calculate $\iint_R 2x^2y\,dA$ where $R$ is the region in the first quadrant bounded below by $y=\sqrt{x}$ and above by the line $y=1$.



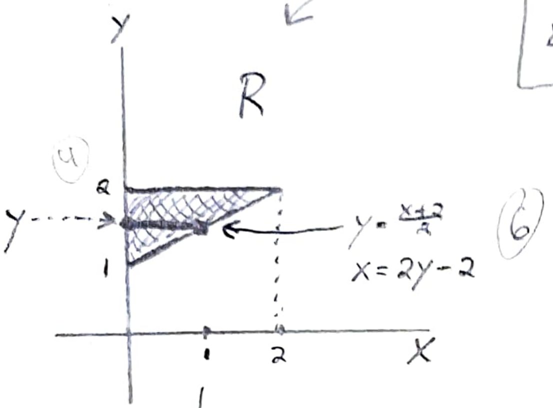

$$R = \boxed{\begin{array}{c} 0 \le x \le 1 \\ \sqrt{x} \le y \le 1 \end{array}}$$ (4)

$$\iint_R 2x^2y\,dA = \int_0^1\int_{\sqrt{x}}^1 2x^2y\,dy\,dx = \int_0^1 x^2y^2\Big]_{y=\sqrt{x}}^{y=1} dx = \int_0^1 x^2(1-x)\,dx$$ (4)

$$= \int_0^1 x^2 - x^3\,dx = \frac{1}{3}x^3 - \frac{1}{4}x^4\Big]_{x=0}^{x=1} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$ (4) (1)

16 pts (2) Reverse the order of integration in the iterated integral $\int_0^2 \int_{\frac{x+2}{2}}^2 f(x,y)\,dy\,dx$. Of course, you cannot calculate the iterated integral.

$R =$ $\boxed{\begin{array}{c} 0 \le x \le 2 \\ \frac{x+2}{2} \le y \le 2 \end{array}}$

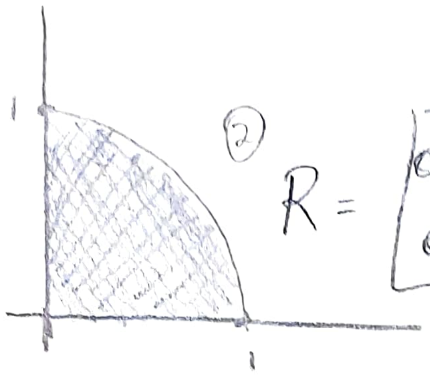



$R =$ $\boxed{\begin{array}{c} 1 \le y \le 2 \\ 0 \le x \le 2y-2 \end{array}}$ 6

4

$$\int_0^2 \int_{\frac{x+2}{2}}^2 f(x,y)\,dy\,dx = \boxed{\int_1^2 \int_0^{2y-2} f(x,y)\,dx\,dy}$$

4pts (3) Use polar coordinates to calculate $\iint_R x + y\, dA$ in which $R$ is the portion in the first quadrant of the disk of radius 1 centered at the origin.



(2)

$$R = \boxed{\begin{array}{l} 0 \le \theta \le \frac{\pi}{2} \\ 0 \le r \le 1 \end{array}}$$ (3)

$$\iint_R x+y\, dA = \int_0^{\frac{\pi}{2}}\int_0^{1} (r\cos(\theta) + r\sin(\theta))\, r\, dr\, d\theta$$

(4) (2) (2)

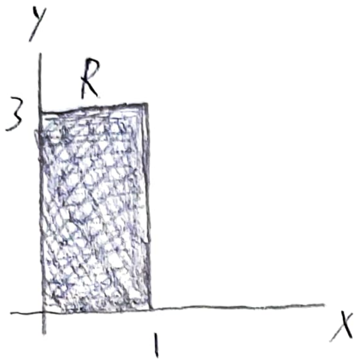
$$= \int_0^{\frac{\pi}{2}}\int_0^{1} r^2(\cos(\theta)+\sin(\theta))\, dr\, d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{3}r^3(\cos(\theta)+\sin(\theta))\Big]_{r=0}^{r=1} d\theta$$

$$= \frac{1}{3}\int_0^{\frac{\pi}{2}} \cos(\theta)+\sin(\theta)\, d\theta = \frac{1}{3}(\sin(\theta) - \cos(\theta))\Big]_{\theta=0}^{\theta=\frac{\pi}{2}}$$

(4)

$$= \frac{1}{3}(1 - 0 - (0-1)) = \frac{2}{3}$$

16pts (4) Let $R$ be the rectangular region in the $xy$-plane defined by $0 \leq x \leq 1$ and $0 \leq y \leq 3$. Say that $R$ has density function $f(x, y)$ units mass per unit area. Set up four iterated integrals measuring: the mass of $R$, the center of mass of $R$, and the average value of $f(x, y)$ on $R$. Of course, you cannot calculate these iterated integrals.



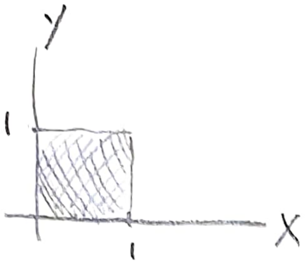

$$\text{Mass}(R) = \iint_R f(x,y)\,dA \quad 5$$

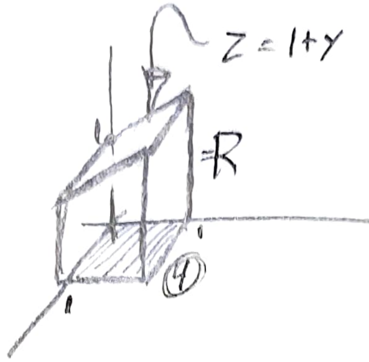
$$\text{Avg value } f(x,y) = \frac{1}{\text{Area}(R)} \iint_R f(x,y)\,dA = \frac{1}{3} \iint_R f(x,y)\,dA \quad 5$$

$$\bar{x} = \frac{1}{\text{mass}(R)} \iint_R x\, f(x,y)\,dA \quad 3$$

$$\bar{y} = \frac{1}{\text{mass}(R)} \iint_R y\, f(x,y)\,dA \quad 3$$

2pts (5) Calculate $\iiint_R x^3y\,dV$ where $R$ is the region in the first octant that is under the plane $z = 1 + y$ and above the square in the xy-plane given by $0 \le x \le 1$ and $0 \le y \le 1$.




$$\boxed{\begin{array}{l} 0 \le x \le 1 \\ 0 \le y \le 1 \\ 0 \le z \le 1+y \end{array}}$$

4

$$\iiint_R x^3y\,dV = \int_0^1\int_0^1\int_0^{1+y} x^3y\,dz\,dy\,dx = \int_0^1\int_0^1 x^3y(1+y)\,dy\,dx = \int_0^1\int_0^1 x^3(y+y^2)\,dy\,dx$$

4

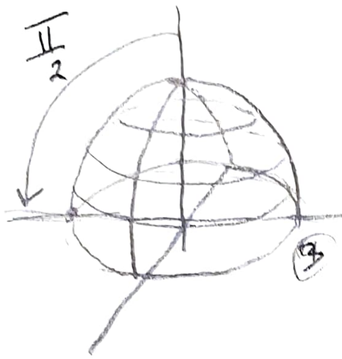
$$= \int_0^1 x^3\left(\frac{1}{2}y^2+\frac{1}{3}y^3\right)\Big]_{y=0}^{y=1} dx = \int_0^1 x^3\left(\frac{1}{2}+\frac{1}{3}\right)dx = \frac{5}{6}\int_0^1 x^3\,dx = \frac{5}{24}x^4\Big]_{x=0}^{x=1}$$

$$= \frac{5}{24}$$

5

17pts

(6) Use spherical coordinates to calculate $\iiint_R x^2 + y^2 + z^2 \, dV$ where $R$ is the half sphere of radius 1 centered at the origin and above the $xy$-plane.




$$R = \boxed{\begin{array}{l} 0 \le \varphi \le \frac{\pi}{2} \\ 0 \le \theta \le 2\pi \\ 0 \le \rho \le 1 \end{array}} \quad (4)$$

$$\iiint_R x^2+y^2+z^2 \, dV = \int_0^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^1 \underset{3}{\boxed{\rho^2}}\,\underset{3}{\boxed{\rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi}} = \int_0^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^1 \rho^4 \sin(\varphi)\, d\rho\, d\theta\, d\varphi$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{2\pi} \frac{1}{5}\rho^5 \sin(\varphi)\Big]_{\rho=0}^{\rho=1} d\theta\, d\varphi = \frac{1}{5}\int_0^{\frac{\pi}{2}}\int_0^{2\pi} \sin(\varphi)\, d\theta\, d\varphi = \frac{2\pi}{5}\int_0^{\frac{\pi}{2}} \sin(\varphi)\, d\varphi$$

$$= \frac{2\pi}{5}(-\cos(\varphi))\Big]_{\varphi=0}^{\varphi=\frac{\pi}{2}} = \frac{2\pi}{5}(0-(-1)) = \frac{2\pi}{5} \quad (8)$$