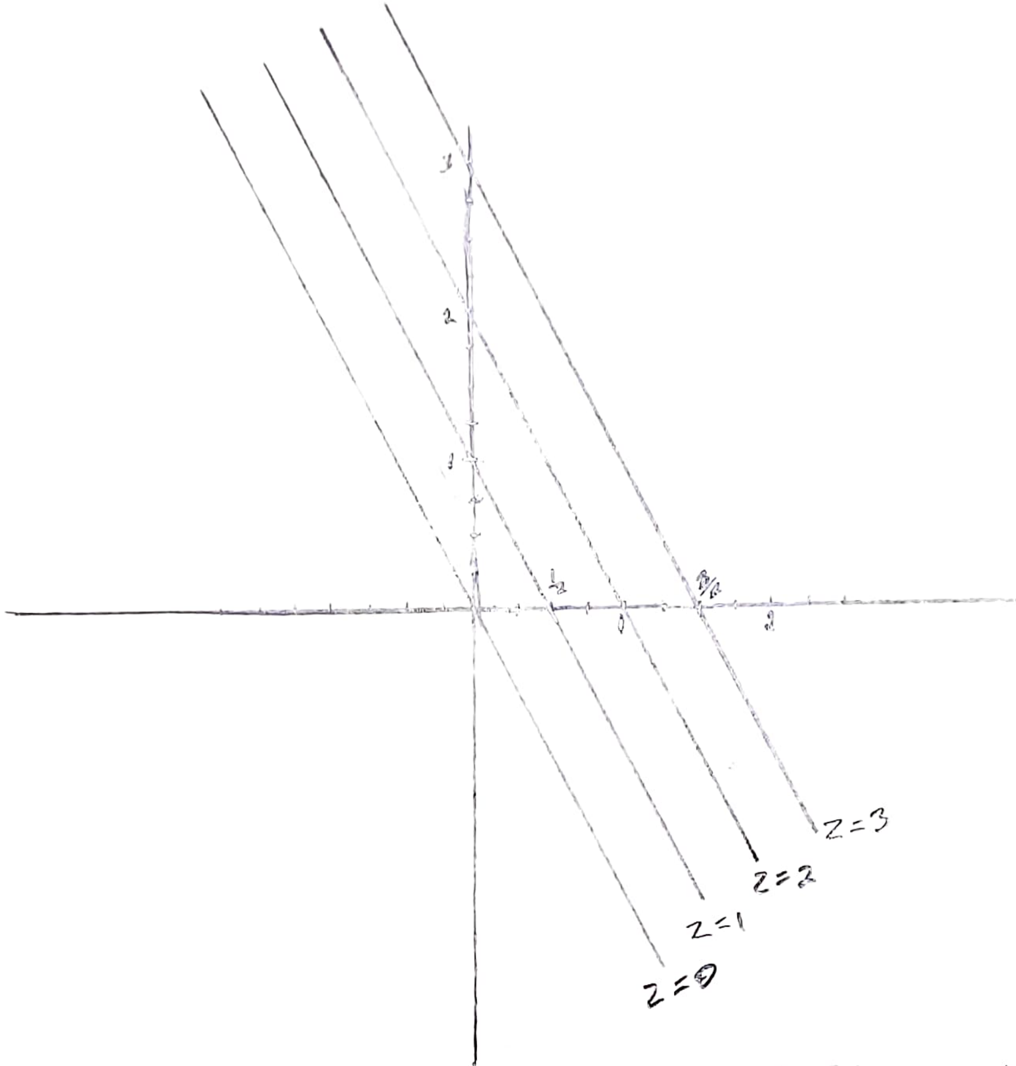
Math2320 – Fall 2022 – Exam 1 Name:__________

16 pts

(1) On the same set of coordinate axes, sketch the level curves for the function $f(x, y) = 2x + y$ for $z = 0, 1, 2, 3$. Label all of the intersection points with the coordinate axes.



$0 = 2x + y \rightarrow y = -2x$

$1 = 2x + y \rightarrow y = -2x + 1$

$2 = 2x + y \rightarrow y = -2x + 2$

$3 = 2x + y \rightarrow y = -2x + 3$

16 pts

(2) Find all of the first and second partial derivatives of $f(x, y) = \dfrac{x^3}{y} + e^{x^2+y^2}$.

$$\boxed{f_x = \frac{3x^2}{y} + 2xe^{x^2+y^2}}$$ 5

$$\boxed{f_{xy} = \frac{-3x^2}{y^2} + 4xye^{x^2+y^2}}$$ 3

notation
1

$$f_{xx} = \frac{6x}{y} + 2e^{x^2+y^2} + 4x^2e^{x^2+y^2}$$

$$\boxed{f_{xx} = \frac{6x}{y} + (2+4x^2)e^{x^2+y^2}}$$ 3

$$\boxed{f_y = \frac{-x^3}{y^2} + 2ye^{x^2+y^2}}$$ 3

$$\boxed{f_{yy} = \frac{2x^3}{y^3} + (2+4y^2)e^{x^2+y^2}}$$ 3

17 pts

(3) Given $f(x,y) = \ln(x^2+y^2)$, calculate $\nabla f(x,y)$, $\nabla f(1,2)$, and $D_{\langle \frac{5}{13}, -\frac{12}{13}\rangle} f(1,2)$. What is the maximum possible value among directional derivatives of $f(x,y)$ at $(1,2)$?

$f_x = \dfrac{2x}{x^2+y^2}$ $\qquad f_x(1,2) = \dfrac{2}{5}$

$f_y = \dfrac{2y}{x^2+y^2}$ $\qquad f_y(1,2) = \dfrac{4}{5}$

$\boxed{\nabla f(x,y) = \left\langle \dfrac{2x}{x^2+y^2}, \dfrac{2y}{x^2+y^2} \right\rangle}$ 2 $\quad \boxed{\nabla f(1,2) = \left\langle \dfrac{2}{5}, \dfrac{4}{5} \right\rangle}$ 2

$D_{\vec{u}} f(1,2) = \nabla f \cdot \vec{u}$ 2 $= \left\langle \dfrac{2}{5}, \dfrac{4}{5} \right\rangle \cdot \left\langle \dfrac{5}{13}, \dfrac{-12}{13} \right\rangle = \dfrac{10-48}{65} = \boxed{\dfrac{-38}{65}}$ 3

Max directional derivative $= |\nabla f(1,2)|$ 4 $= \sqrt{\dfrac{4+16}{25}} = \boxed{\dfrac{2\sqrt{5}}{5} = \dfrac{2}{\sqrt{5}}}$ 3

12pts (4) Find the equations for the tangent plane and the normal line to the surface $z = x + x^2y^3$ at the point $(-2, 1)$.

$Z_x = 1 + 2xy^3$ $\quad$ $Z_y = 3x^2y^2$ $\quad$ $Z(-2,1) = -2+4 = 2$

5

$Z_x(-2,1) = 1 - 4 = -3$ $\quad$ $Z_y(-2,1) = 12$

Tangent Plane

$$Z - Z(-2,1) = Z_x(-2,1)(x+2) + Z_y(-2,1)(y-1) \quad 2$$

$$\boxed{Z - 2 = -3(x+2) + 12(y-1)} \quad 4$$

Normal line

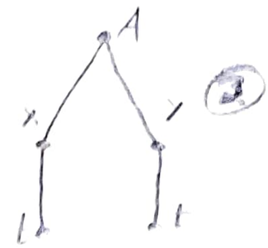
$$\vec{n} = \langle Z_x, Z_y, -1 \rangle = \langle -3, 12, -1 \rangle \quad 2$$

$$\boxed{\begin{aligned} x &= -2 - 3t \\ y &= 1 + 12t \\ z &= 2 - t \end{aligned}} \quad 4$$

12 pts

(5) A rectangular box has length x, width x, and height y. Suppose that x and y are functions of time parameter t. Consider the surface area of the box $A = 2x^2 + 4xy$.

(a) Use the chain rule to find an expression for $\frac{dA}{dt}$.



3 $$\frac{dA}{dt} = \frac{\partial A}{\partial x}\frac{dx}{dt} + \frac{\partial A}{\partial y}\frac{dy}{dt}$$

3 $$\boxed{\frac{dA}{dt} = (4x+4y)\frac{dx}{dt} + (4x)\frac{dy}{dt}}$$

(b) Suppose that x is increasing at a rate of 0.2 inches/second, y is decreasing at a rate of 0.1 inches/second. Find $\frac{dA}{dt}$ when $(x, y) = (1, 2)$. Is the surface area increasing or decreasing at this point?

$$\left.\frac{dA}{dt}\right|_{(x,y)=(1,2)} = 12(0.2) + 4(-0.1) = 2\ \frac{\text{inches}^2}{\text{second}}$$ 3

3

A is increasing 3

17 pts

(6) Find the critical point(s) for the function $f(x, y) = 2x^2 + 2xy + y^2 + 2x - y$. Determine whether the critical point is a relative minimum, relative maximum, or neither.

(6)

$f_x = 4X + 2Y + 2 = 0$

$f_y = 2X + 2Y - 1 = 0$

$f_{xx} = 4$ $\quad f_{xy} = f_{yx} = 2$

$f_{yy} = 2$

$2X + Y = -1$

$-(2X + 2Y = 1)$

$-Y = -2$

$\boxed{Y = 2}$

$2X + 4 = 1$

$2X = -3$

$\boxed{X = \frac{-3}{2}}$

(3)

$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix} = 4 > 0$

So $D > 0$ and $f_{xx} > 0$ (4)

So $\left(\frac{-3}{2}, 2\right)$ is a relative minimum

(4) Only critical point

$\boxed{(X, Y) = \left(-\frac{3}{2}, 2\right)}$