No class Monday 9/5 for Labor-day holiday.

For Wednesday 9/7 discussion problems

2.6 319–330 sketch the surface given by the equation shown.

2.7 problems on syllabus.

2.4

(184) Calculate $\vec{u} \times \vec{v}$ where $\vec{u} = \langle 3, 2, -1 \rangle = 3\hat{i} + 2\hat{j} - \hat{k}$

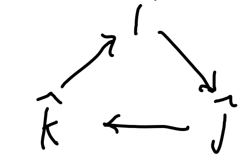
$\vec{v} = \langle 1, 1, 0 \rangle = \hat{i} + \hat{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ 1 & 1 & 0 \end{vmatrix} = \left\langle \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix}, -\begin{vmatrix} 3 & -1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} \right\rangle$$

$$= \langle 0-(-1), -(0-(-1)), 3-2 \rangle$$

$$= \langle 1, -1, 1 \rangle$$

another method



$\hat{i}\times\hat{i}=0$
$\hat{j}\times\hat{j}=0$
$\hat{k}\times\hat{k}=0$

$(3\hat{i}+2\hat{j}-\hat{k})\times(\hat{i}+\hat{j}) =$ ← distribution

$3i\times i + 3i\times j + 2j\times i + 2j\times j - k\times i - k\times j = 3k - 2k - j + i = i - j + k$

($3i\times i \to 0$, $2j\times j \to 0$)

$= \langle 1,-1,1\rangle$

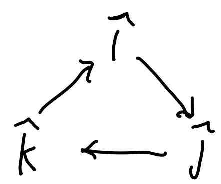
4-dimensional unit vectors

$1 = \langle 1\ 0\ 0\ 0\rangle$

$\hat{i} = \langle 0, 1, 0\ 0\rangle$

$\hat{j} = \langle 0\ 0\ 1\ 0\rangle$

$\hat{k} = \langle 0\ 0\ 0\ 1\rangle$



$\hat{i}\times\hat{i}=-1$
$\hat{j}\times\hat{j}=-1$
$\hat{k}\times\hat{k}=-1$

$1\times\hat{i}=\hat{i}\times 1=\hat{i}$
etc...

(199) $\vec{u}=\langle -1, 0, e^t\rangle \quad \vec{v}=\langle 1, e^{-t}, 0\rangle$

find $\vec{w}$ orthogonal to both $\vec{u}$ and $\vec{v}$.

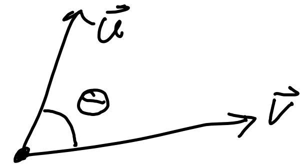
Of course we can choose $\vec{w}=\vec{u}\times\vec{v}$ or $k(\vec{u}\times\vec{v})$ for any scalar k.

$$\vec{w}=\vec{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 0 & e^t \\ 1 & e^{-t} & 0\end{vmatrix} = \langle -1, e^t, -e^{-t}\rangle$$

Check $\langle -1, e^t, -e^{-t}\rangle\cdot\langle -1, 0, e^t\rangle = 1+0-1=0$ ✓

$\langle -1, e^t, -e^{-t}\rangle\cdot\langle 1, e^{-t}, 0\rangle = -1+1+0=0$

(204) $\vec{u} = \langle -1, 3, 1 \rangle \qquad \vec{V} = \langle 1, -2, 0 \rangle$



$$\vec{u} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & 1 \\ 1 & -2 & 0 \end{vmatrix} = \langle 2, 1, -1 \rangle$$

a property cross products

$$|\vec{u} \times \vec{V}| = |\vec{u}||\vec{V}| \sin(\theta)$$

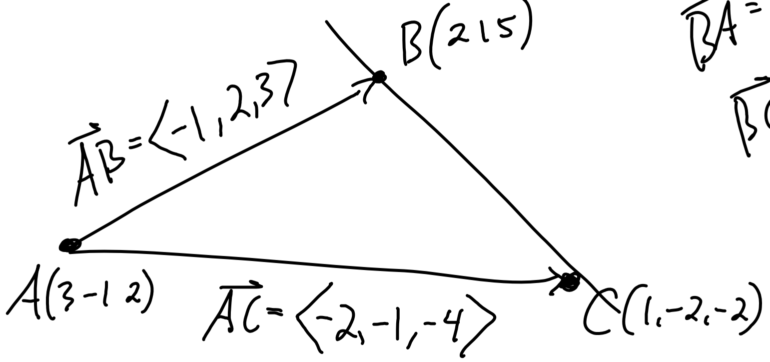
$$\sqrt{6} = \sqrt{11}\sqrt{5} \sin(\theta)$$

$$\sqrt{\frac{6}{55}} = \sin(\theta)$$

$$\text{Arcsin}\left(\sqrt{\frac{6}{55}}\right) = \theta$$

$$19.29° \approx \theta$$

(211)



$\vec{BA} = \langle 1, -2, -3 \rangle$

$\vec{BC} = \langle -1, -3, -7 \rangle$

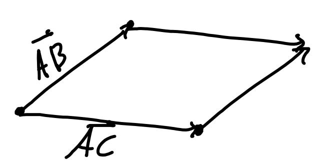
$$\frac{|BA \times BC|}{|BC|} =$$

$$\begin{vmatrix} i & j & k \\ 1 & -2 & -3 \\ -1 & -3 & -7 \end{vmatrix} = \langle 5, 10, -5 \rangle$$

$$\frac{5\sqrt{6}}{\sqrt{59}}$$

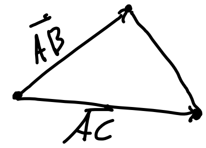
$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ -2 & -1 & -4 \end{vmatrix} = \langle -5, -10, 5 \rangle$$

$$|\vec{AB} \times \vec{AC}| = 5 \langle 1, 2, 1 \rangle = 5\sqrt{6}$$

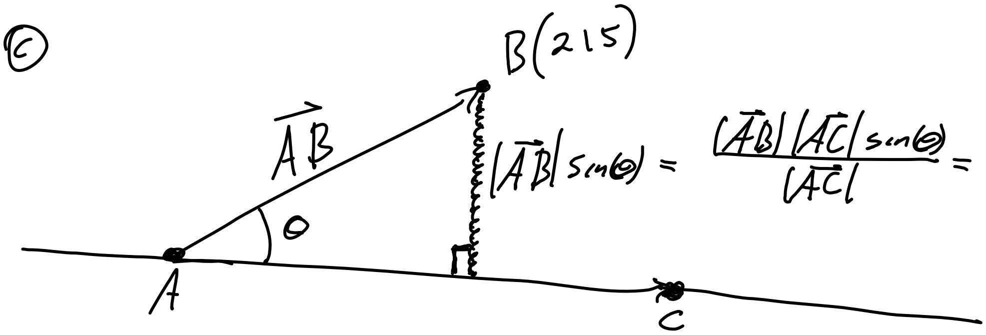


(a) Area of parallelogram defined by $\vec{AB}$ and $\vec{AC}$ $= 5\sqrt{6}$

(b) Area of triangle defined by $\vec{AB}$ and $\vec{AC}$ $= \frac{5}{2}\sqrt{6}$

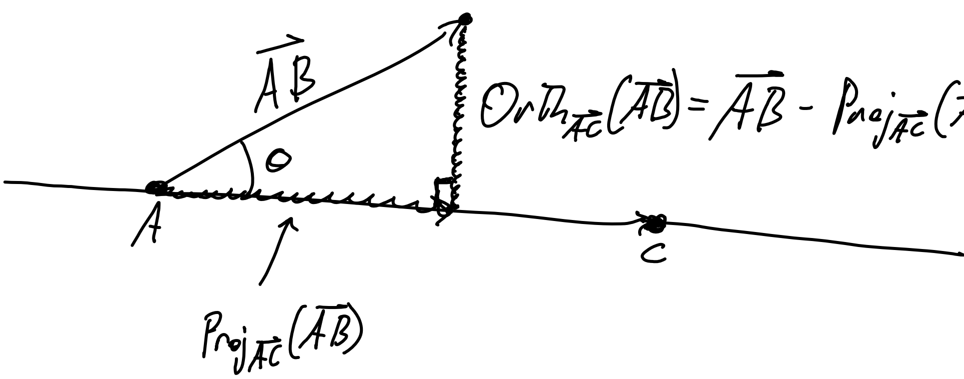


(c)



$$|\vec{AB}|\sin(\theta) = \frac{|\vec{AB}||\vec{AC}|\sin(\theta)}{|\vec{AC}|} = \frac{|\vec{AB}\times\vec{AC}|}{|\vec{AC}|} = \frac{5\sqrt{6}}{\sqrt{21}} = 5\sqrt{\frac{6}{21}}$$

<u>Another solution</u>



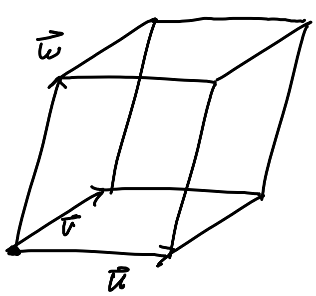
$$\text{Orth}_{\vec{AC}}(\vec{AB}) = \vec{AB} - \text{Proj}_{\vec{AC}}(\vec{AB})$$

distance we want is

$$|\text{Orth}_{\vec{AC}}(\vec{AB})|$$

---

(214)



Volume of this "parallelopiped" is $|\vec{u}\cdot(\vec{v}\times\vec{w})|$ which is the absolute value of the 3×3 determinant

$$\begin{vmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \end{vmatrix}$$

In this example

$\vec{u} = \langle -3 \;\; 5 \;\; -1 \rangle$

$\vec{v} = \langle 0 \;\; 2 \;\; -2 \rangle$

$\vec{w} = \langle 3 \;\; 1 \;\; 1 \rangle$

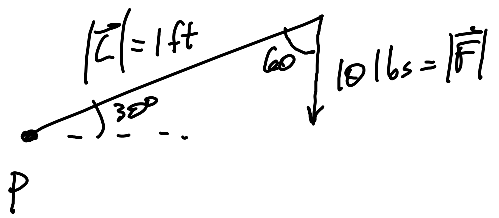
So volume $= \left| \begin{vmatrix} -3 & 5 & -1 \\ 0 & 2 & -2 \\ 3 & 1 & 1 \end{vmatrix} \right| = \left| -3 \begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix} - 5 \begin{vmatrix} 0 & -2 \\ 3 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix} \right|$

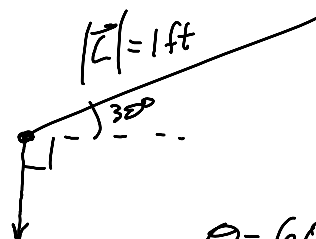
$= \left| -3(4) - 5(6) - (-6) \right|$

$= \left| -12 - 30 + 6 \right| = \boxed{36}$

(221) omit

(235)





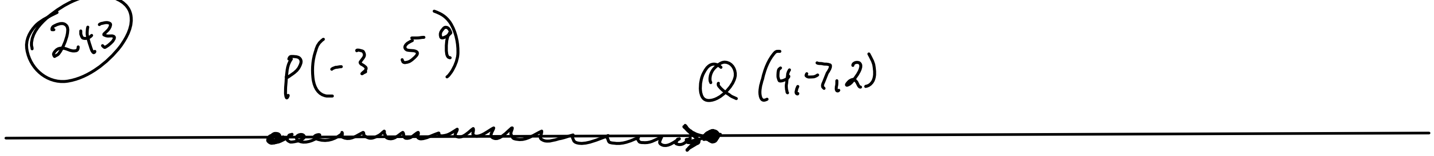
$\theta = 60$ or 120

Torque $= |\vec{L}||\vec{F}|\sin(\theta)$

$= 1 \cdot 10 \sin(60) = \boxed{5\sqrt{3} \text{ ft} \cdot \text{lbs}}$

(2.5)

(243)



$\vec{PQ} = \langle 7, -12, -7 \rangle$

vector equation $\langle x, y, z \rangle = \langle -3, 5, 9 \rangle + t\langle 7, -12, -7 \rangle$

parametric equations

$$x = -3 + 7t$$
$$y = 5 - 12t$$
$$z = 9 - 7t$$

Line segment from P to Q

$x = -3 + 7t$
$y = 5 - 12t$ with $0 \le t \le 1$
$z = 9 - 7t$

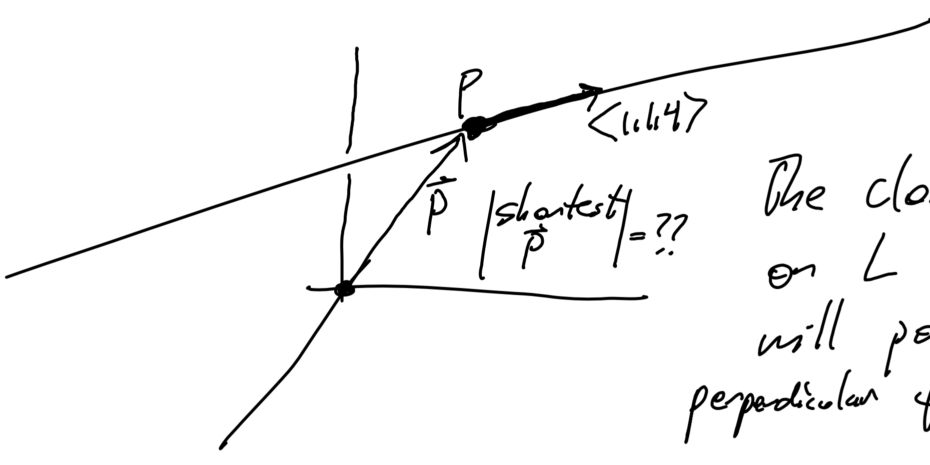
symmetric equations

$$\frac{x+3}{7} = \frac{y-5}{-12} = \frac{z-9}{-7}$$

(251) L: $x = 1 + t$, $y = 3 + t$, $z = 5 + 4t$

a point on the line $(1, 3, 5)$

slope vector $\langle 1, 1, 4 \rangle$



The closest point P on L to $(0,0,0)$ will position vector $\vec{P}$ perpendicular to $\langle 1, 1, 4 \rangle$

$\vec{P} = \langle x, y, z \rangle$ and $\langle x, y, z \rangle \cdot \langle 1, 1, 4 \rangle = 0$ and $\langle x, y, z \rangle$ is on L.

Since $\langle x, y, z\rangle$ is on L $\langle x, y, z\rangle = \langle 1+t, 3+t, 5+4t\rangle$ and

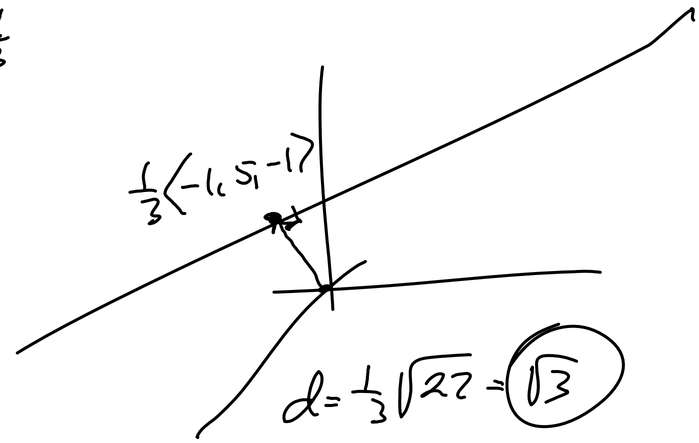
$$\langle 1+t, 3+t, 5+4t\rangle \cdot \langle 1,1,4\rangle = 0$$

$$1+t+3+t+4(5+4t) = 0$$

$$24+18t = 0$$

$$t = \frac{-24}{18} = \frac{-4}{3}$$

$$\langle x, y, z\rangle = \left\langle -\tfrac{1}{3}, \tfrac{5}{3}, -\tfrac{1}{3}\right\rangle$$



$d = \frac{1}{3}\sqrt{27} = \sqrt{3}$

(255)

$$\begin{aligned} x &= 1+t \\ y &= t \\ z &= 2+t \end{aligned}$$ L_1

$$x-3 = y-1 = z-3$$ L_2

slope $= \langle 1\ 1\ 1\rangle$

2 points on L_1

(1 0 2)

(2, 1, 3)

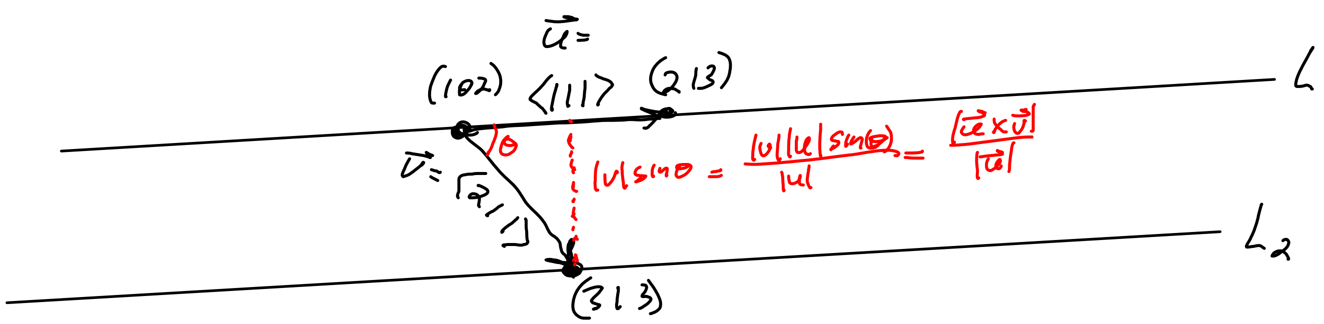
$$\begin{aligned} x &= 3+t \\ y &= 1+t \\ z &= 3+t \end{aligned}$$

slope $= \langle 1\ 1\ 1\rangle$

a point on L_2

(3, 1, 3)

lines are parallel



$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \langle 0, 1, -1 \rangle$$

$|\vec{u}\times\vec{v}| = \sqrt{2}$

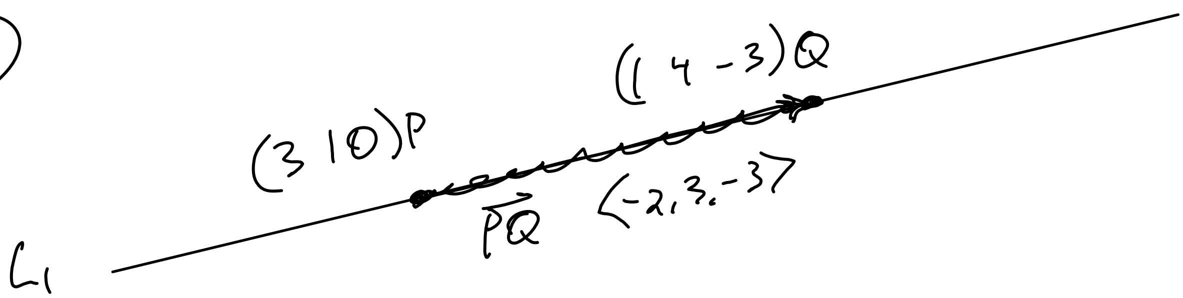
$|\vec{u}| = \sqrt{3}$

distance L_1 to L_2 $= \dfrac{|\vec{u}\times\vec{v}|}{|\vec{u}|} = \boxed{\sqrt{\tfrac{2}{3}}}$

(256) slope vectors $\langle 0, 0, 1 \rangle$ and $\langle 0, 0, -3 \rangle$

So they are parallel. Find the distance on your own.

(257)



L_2 has slope $\langle 3, 8, 6 \rangle$

Because $\langle -2, 3, -3 \rangle \cdot \langle 3, 8, 6 \rangle = -6 + 24 - 18 = 0$

This means the lines w/ these slopes are perpendicular.

(261) L_1 $x = y - 1 = -z$ $\qquad$ L_2 $x - 2 = -y = \frac{z}{2}$

$x = t$ $\qquad$ $x = 2 + t$

$y = 1 + t$ $\qquad$ $y = -t$

$z = -t$ $\qquad$ $z = 2t$

slope $= \langle 1, 1, -1 \rangle$ $\qquad$ slope $= \langle 1, -1, 2 \rangle$

L_1 and L_2 are skew or intersecting.

If L_1 and L_2 intersect, then there is a solution to

$t = 2 + s \longrightarrow t - s = 2$

$1 + t = -s$

$-t = 2s \longrightarrow t = -2s$

$-2s - s = 2$

$-3s = 2$

$\boxed{s = -\frac{2}{3}}$

$\boxed{t = \frac{4}{3}}$

$1 + t = -s$

$1 + \frac{4}{3} \neq \frac{2}{3}$ so L_1 and L_2 are skew because there is no intersection point.

(264)

L_1

$3x = y + 1 = 2z$

$x = \frac{1}{3}t$

$y = -1 + t$

$z = \frac{1}{2}t$

slope $= \langle \frac{1}{3}, 1, \frac{1}{2} \rangle$

L_2

$6 + 2t = x$

$12 + 6t = y$

$9 + 3t = z$

slope $= \langle 2, 6, 3 \rangle$

parallel or equal.

L_1 contains the point $(0, -1, 0)$. Thus $L_2 = L_1$ if and only if L_2 also contains $(0, -1, 0)$.

$x = 6 + 2t = 0$

$t = -3$

using $t = -3$ $(x, y, z) = (0, -1, 0)$