Discussion Problems 2.4, 2.5 for next time

Sec 2.3

125 $\langle 2,2,-1\rangle \cdot \langle -1,2,2\rangle = (2)(-1) + (2)(2) + (-1)(2) = 0$

By the way $\vec{u}\cdot\vec{v} = 0$ if and only if $\vec{u}, \vec{v}$ are perpendicular.

129 $\vec{a} = \langle 1,1,0\rangle$, $\vec{b} = \langle 1,0,-1\rangle$, $\vec{c} = \langle -1,2,-4\rangle$.

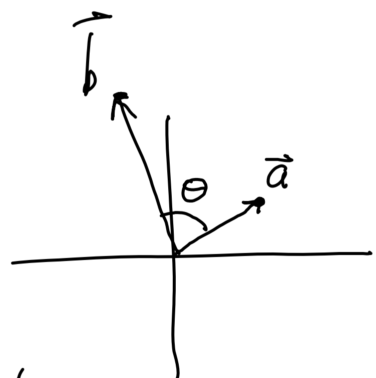
$\vec{a}\cdot\vec{b} = \langle 1\ 1\ 0\rangle \cdot \langle 1\ 0\ -1\rangle = 1$

$(\vec{a}\cdot\vec{b})\vec{c} = \langle -1,2,-4\rangle$

$\vec{a}\cdot\vec{c} = \langle 1\ 1\ 0\rangle \cdot \langle -1\ 2\ -4\rangle = -1+2+0 = 1$

$(\vec{a}\cdot\vec{c})\vec{b} = \langle 1,0,-1\rangle$

132 $\vec{a} = \langle 2,1\rangle$, $\vec{b} = \langle -1,3\rangle$



Remember

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos(\theta)$ by definition.

Thus $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \cos(\theta)$

$\dfrac{-2+3}{\sqrt{4+1}\sqrt{1+9}} = \dfrac{1}{\sqrt{50}} = \dfrac{1}{5\sqrt{2}} = \cos(\theta)$

since $\vec{a}\cdot\vec{b} > 0$
θ is acute

Thus $\theta = \arccos\left(\frac{1}{5\sqrt{2}}\right)$

$\theta \approx 1.4288$ radians $\approx 81.87°$

(135) $\vec{u} = \langle 3, -1, 2 \rangle$ $\quad \vec{v} = \langle 1, -1, -2 \rangle$

$$\cos(\theta) = \frac{u \cdot v}{|u||v|} = \frac{3+1-4}{|u||v|} = 0 \qquad \theta = 90° = \frac{\pi}{2}$$

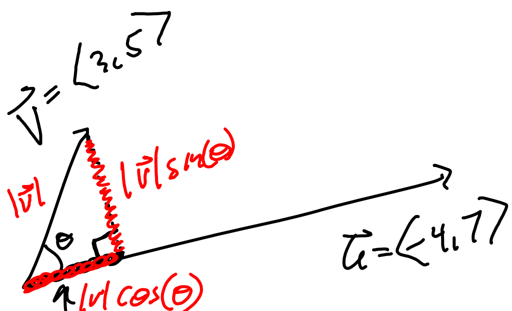
(142) $\vec{a} = \langle x, x \rangle$ $\quad \vec{b} = \langle -y, y \rangle$ are $\vec{a}$ and $\vec{b}$ orthogonal?

$\vec{a} \cdot \vec{b} = \langle x, x \rangle \cdot \langle -y, y \rangle = -xy + xy = 0$ so yes they are orthogonal.

(143) $\vec{a} = \langle 3, -1, -2 \rangle$ $\quad \vec{b} = \langle -2, -3, 1 \rangle$

$\vec{a} \cdot \vec{b} = -6 + 3 - 2 = -5$ so no, $\vec{a}$ and $\vec{b}$ are not orthogonal.

(168)



length of this segment is the called the <u>scalar projection</u> of $\vec{v}$ onto $\vec{u}$. OR the <u>component of $\vec{v}$ in the direction of $\vec{u}$.</u>

Scalar projection $= |\vec{v}|\cos(\theta) = \cancel{|v|}\dfrac{\vec{u}\cdot\vec{v}}{|u|\cancel{|v|}} = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|} = \dfrac{-12+35}{\sqrt{16+49}} = \boxed{\dfrac{23}{\sqrt{65}}}$

Now

$$\text{proj}_{\vec{u}}(\vec{v}) = \underbrace{\frac{23}{\sqrt{65}}}_{\text{length } \frac{23}{\sqrt{65}}}\underbrace{\left(\frac{1}{|\vec{u}|}\vec{u}\right)}_{\text{unit vector length}=1} = \frac{23}{\sqrt{65}}\left(\frac{1}{\sqrt{65}}\langle -4, 7\rangle\right) = \boxed{\left\langle -\frac{92}{65}, \frac{161}{65}\right\rangle}$$

OR use the formula

$$\text{proj}_{\vec{u}}(\vec{v}) = \frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}\vec{u} = \frac{23}{65}\langle -4, 7\rangle = \boxed{\left\langle -\frac{92}{65}, \frac{161}{65}\right\rangle}$$

Scalar projection $= \left|\left\langle \frac{-92}{65}, \frac{161}{65}\right\rangle\right| = \boxed{\dfrac{23}{\sqrt{65}}}$.

169 $\quad \vec{u} = \langle 3\ 0\ 2\rangle \quad \vec{v} = \langle 0\ 2\ 4\rangle$

Scalar projection of $\vec{v}$ onto $\vec{u}$ $= \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|} = \dfrac{8}{\sqrt{13}}$

$$\text{proj}_{\vec{u}}(\vec{v}) = \frac{8}{\sqrt{13}}\left(\frac{1}{\sqrt{13}}\langle 3, 0, 2\rangle\right) = \left\langle \frac{24}{13}, 0, \frac{16}{13}\right\rangle$$

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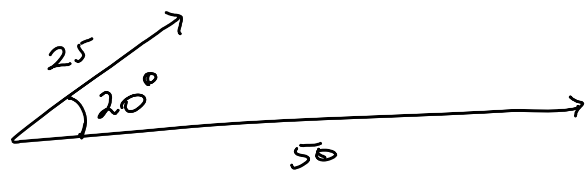
Work $= \vec{F} \cdot \vec{d} = |\vec{F}||\vec{d}|\cos(\theta) = (25)(50)\cos(20^\circ) = 1174$ ft·lbs.



180 omit.