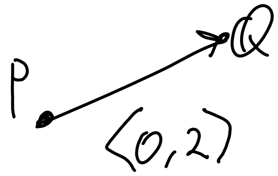
# Discussion Problems

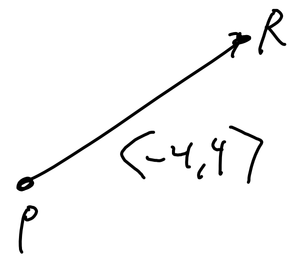
Sections 2.3–2.4 on the syllabus.

$P(1,3) \qquad Q(1,5) \qquad R(-3,7)$

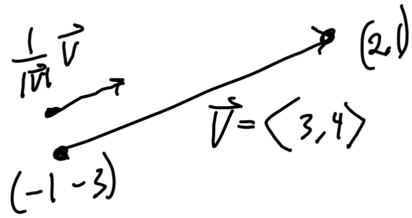
1.




2.




11.




So $|\vec{v}| = \sqrt{3^2+4^2} = \sqrt{25} = 5$

So a unit vector in the same direction as $\vec{v}$ is

$$\frac{1}{|\vec{v}|}\vec{v} = \frac{1}{5}\langle 3,4\rangle = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

16. $\vec{a} = 2\hat{\imath} = \langle 2,0\rangle$

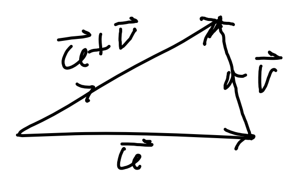
$\vec{b} = 2\hat{\imath} - 2\hat{\jmath} = \langle -2,2\rangle$

So $\vec{a}+\vec{b} = \langle 0,2\rangle = 2\hat{\jmath}$

$\vec{a}-\vec{b} = \langle 4,-2\rangle = 4\hat{\imath} - 2\hat{\jmath}$

The triangle inequality

$$|\vec{u}+\vec{v}| \le |\vec{u}| + |\vec{v}|$$




$|\vec{a}| = 2$

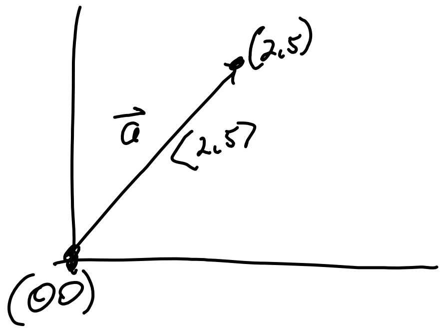
$|\vec{b}| = \sqrt{4+4} = 2\sqrt{2}$

$|\vec{a}+\vec{b}| = 2$

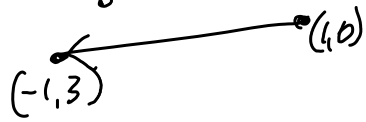
So we see that $2 \le 2 + 2\sqrt{2}$.

$$2\vec{a}-\vec{b}=\langle 4,0\rangle-\langle -2,2\rangle = \langle 6,-2\rangle.$$

(18)




$\vec{b}=\langle -2,3\rangle$




$$|\vec{a}-3\vec{b}+14\hat{i}-14\hat{j}| =$$

$$|\langle 2,5\rangle - 3\langle -2,3\rangle + \langle 14,14\rangle| =$$

$$|\langle 22,10\rangle| = 2|\langle 11,5\rangle| = 2\sqrt{121+25}$$

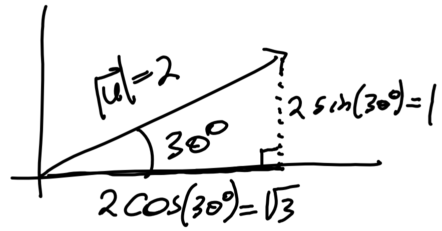
$$= 2\sqrt{146}$$

(25) $|\vec{v}|=7$ direction is same as $\vec{u}=\langle 3,4\rangle$

$|\vec{u}|=\sqrt{3^2+4^2}=\sqrt{25}=5$ $\quad \frac{1}{|\vec{u}|}\vec{u}=\langle \frac{3}{5},\frac{4}{5}\rangle$ has length 1 and same direction as $\vec{u}$.

Therefore $\vec{v}=7\langle \frac{3}{5},\frac{4}{5}\rangle=\langle \frac{21}{5},\frac{28}{5}\rangle$ has length 7 and same direction as $\vec{u}$.

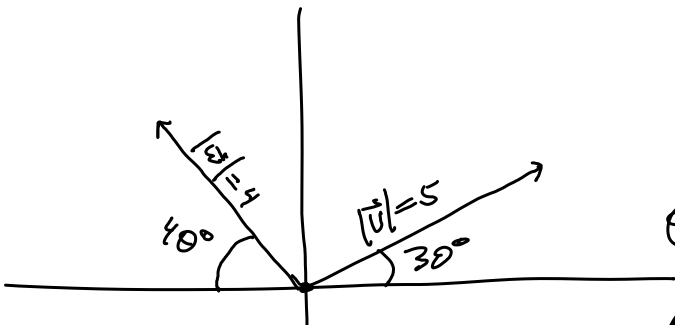
(29) $|\vec{u}| = 2$




Find components of $\vec{u}$.

Therefore $\vec{u} = \langle \sqrt{3}, 1 \rangle$.

For next time try this problem

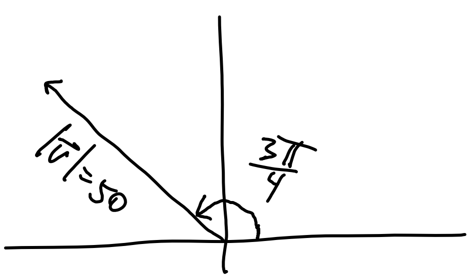



① Find $\vec{v}+\vec{w}$ in component form. Round to 2 decimal places.

② Find $|\vec{v}+\vec{w}|$

③ Find the angle of $\vec{v}+\vec{w}$ with the positive x-axis when the tail of $\vec{v}+\vec{w}$ is at the origin.

(34)

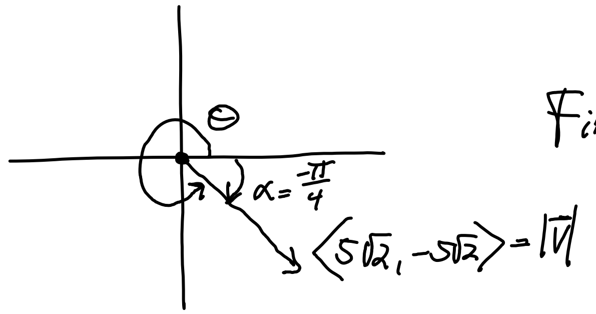



$$\vec{V} = \langle |\vec{v}|\cos(\theta), |\vec{v}|\sin(\theta) \rangle$$

$$= \left\langle 50\cos\left(\tfrac{3\pi}{4}\right), 50\sin\left(\tfrac{3\pi}{4}\right) \right\rangle$$

$$= \langle -25\sqrt{2}, 25\sqrt{2} \rangle$$

(35)




Find (θ).

$$5\sqrt{2} = |\vec{v}|\cos(\theta)$$
$$-5\sqrt{2} = |\vec{v}|\sin(\theta)$$

Therefore

$$-1 = \frac{\cancel{|\vec{v}|}\sin(\theta)}{\cancel{|\vec{v}|}\cos(\theta)}$$

$$-1 = \tan(\theta)$$

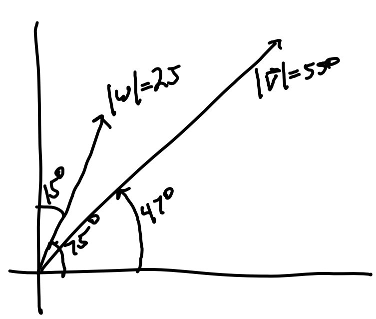
$$\text{Arctan}(-1) = \frac{-\pi}{4}$$

where arctangent gives an answer for an angle $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$

Thus $\theta = \frac{7\pi}{4}$.

(45) omit

(53) The new speed and direction of the airplane relative to the ground is given by the sum of the vectors.




$$\vec{v} = \langle 550\cos(47), 550\sin(47)\rangle$$
$$\vec{w} = \langle 25\cos(75), 25\sin(75)\rangle$$

$$\vec{v} + \vec{w} = \langle 381.57, 426.39\rangle$$

$$|\vec{v} + \vec{w}| = 572.19 \text{ mph}$$

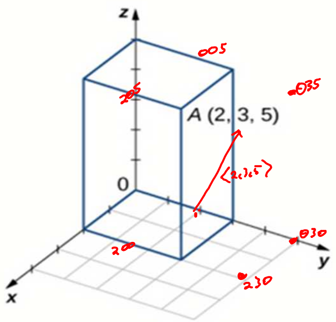
angle for $\vec{v} + \vec{w}$ is $\theta = \text{Arctan}\left(\frac{426.39}{381.57}\right) \approx 48.18°$

Section 2.2 Exercises

61. Consider a rectangular box with one of the vertices at the origin, as shown in the following figure. If point $A(2, 3, 5)$ is the opposite vertex to the origin, then find

a. the coordinates of the other six vertices of the box and

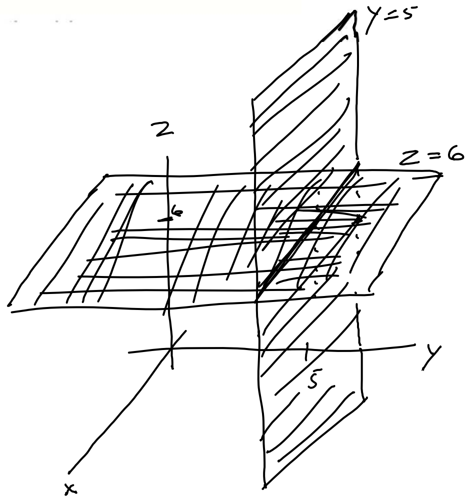
b. the length of the diagonal of the box determined by the vertices $O$ and $A$.



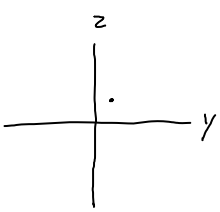

$|\langle 2,3,5\rangle| = \sqrt{4+9+25}$
$= \sqrt{38}$

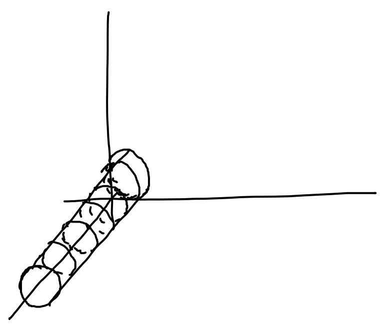
(63) $(y-5)(z-6)=0$

So either $y=5$ or $z=6$




(65) $(y-1)^2+(z-1)^2=1$



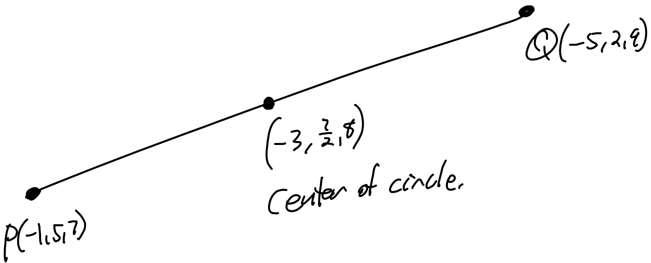



(67) plane contains $(1,1,1)$ parallel to $xy$-plane is $z=1$.

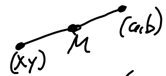
(68) plane contains $(1,-3,2)$ parallel to $xz$-plane is $y=-3$

(70) $x=1$

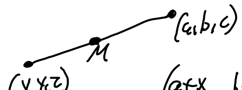
(73) Sphere has diameter




<u>midpoint formula</u>




$$M = \left(\frac{a+x}{2}, \frac{y+b}{2}\right)$$




$$M = \left(\frac{a+x}{2}, \frac{b+y}{2}, \frac{c+z}{2}\right)$$

$$\text{radius} = \frac{1}{2}\sqrt{(-4)^2+(-3)^2+(2)^2}$$
$$= \frac{1}{2}\sqrt{16+9+4}$$
$$= \frac{\sqrt{29}}{2}$$
$$\text{radius}^2 = \frac{29}{4}$$

equation of sphere $\boxed{(x+3)^2+\left(y-\frac{7}{2}\right)^2+(z-8)^2 = \frac{29}{4}}$

(76) example from lecture. Try 75 on your own.

(83) $\vec{a} = \langle -1,-2,4 \rangle \quad \vec{b} = \langle -5,6,-7 \rangle$

$\vec{a}+\vec{b} = \langle -6,4,-3 \rangle$

$4\vec{a} = \langle -4,-8,16 \rangle$

$-5\vec{a}+3\vec{b} = \langle 5,10,-20 \rangle + \langle -15,18,-21 \rangle = \langle -10,28,-41 \rangle$

(87) $\vec{u} = \langle 2,3,4 \rangle \quad \vec{v} = \langle -1,5,-1 \rangle$ find $|\vec{u}-\vec{v}|$ and $|-2\vec{u}|$

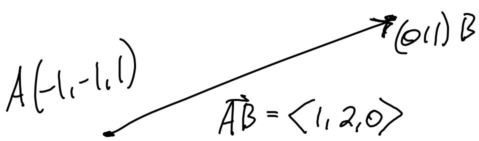
$|\vec{u}-\vec{v}| = |\langle 3,-2,5 \rangle| = \sqrt{9+4+25} = \sqrt{38}$

$|-2\vec{u}| = 2|\vec{u}| = 2\sqrt{4+9+16} = 2\sqrt{29}$

(92) $\vec{u} = \langle 4,-3,6 \rangle \qquad |\vec{u}| = \sqrt{16+9+36} = \sqrt{61}$

unit vector in direction of $\vec{u}$ is $\frac{1}{|\vec{u}|}\vec{u} = \left\langle \frac{4}{\sqrt{61}}, \frac{-3}{\sqrt{61}}, \frac{6}{\sqrt{61}} \right\rangle.$

(104)




$\vec{AB} = \langle 1,2,0 \rangle$

$|\vec{AB}| = \sqrt{5}$

$\vec{u} = \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right\rangle$

$-2\vec{u} = \left\langle \frac{-2}{\sqrt{5}}, \frac{-4}{\sqrt{5}}, 0 \right\rangle$ has length $=2$ points in direction opposite to $\vec{AB}$.

111, 113 omit.