# 3.2 Trigonometric Integrals

Consider $\int \cos^m(x)\sin^n(x)\,dx$

**Case 1:** One of m and n is odd.

$$\int \cos^5(x)\sin^4(x)\,dx = \int \cos^4(x)\sin^4(x)\cos(x)\,dx$$

$$= \int \left(\cos^2(x)\right)^2 \sin^4(x)\cos(x)\,dx = \int \left(1-\sin^2(x)\right)^2 \sin^4(x)\cos(x)\,dx$$

Now let $u = \sin(x)$

$$\frac{du}{dx} = \cos(x)$$

$$\frac{du}{\cos(x)} = dx$$

$$= \int (1-u^2)^2 u^4 \cancel{\cos(x)} \frac{du}{\cancel{\cos(x)}} = \int (1-u^2)^2 u^4\,du$$

$$= \int u^8 - 2u^6 + u^4\,du = \ldots$$

Case II m and n are both even.

Must use the double-angle formulas

$$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$$
$$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$$

example

$$\int \sin^4(x)\,dx = \int (\sin^2(x))^2\,dx = \int \frac{1}{4}(1-\cos(2x))^2\,dx =$$

$$\frac{1}{4}\int 1-2\cos(2x)+\cos^2(2x)\,dx = \frac{1}{4}\int 1-2\cos(2x)+\frac{1}{2}(1+\cos(4x))\,dx$$

$$= \frac{1}{4}\int \frac{3}{2}-2\cos(2x)+\frac{1}{2}\cos(4x)\,dx = \frac{1}{4}\left[\frac{3}{2}x-\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)\right]+C$$

Keep In mind

$$\frac{d}{dx}\sin(kx) = k\cos(kx)$$

so

$$\int \cos(kx)\,dx = \frac{1}{k}\sin(kx)+C$$

Next, consider $\int \sec^m(x)\tan^n(x)\,dx$

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$\sin^2(x)+\cos^2(x)=1$

$\tan^2(x)+1=\sec^2(x)$

Case I $m$ is even (Exponent on $\sec(x)$ is even)

example

$$\int \sec^4(x)\tan^5(x)\,dx = \int \sec^2(x)\tan^5(x)\sec^2(x)\,dx$$

$$= \int (\tan^2(x)+1)\tan^5(x)\sec^2(x)\,dx = \int (u^2+1)u^5\,du = \cdots\cdots$$

let $u=\tan(x)$

$\frac{du}{dx}=\sec^2(x)$

$\frac{du}{\sec^2(x)}=dx$

just a polynomial in $u$.

Case II Odd exponent on $\tan(x)$ and $\sec(x)$ exponent is at least 1.

example

$$\int \sec^3(x)\tan^3(x)\,dx = \int \sec^2(x)\tan^2(x)\sec(x)\tan(x)\,dx$$

$$\int \sec^2(x)(\sec^2(x)-1)\sec(x)\tan(x)\,dx = \int u^2(u^2-1)\,du$$

let $u=\sec(x)$

polynomial in $u$.