Discussion assignment

Exercises for Sections 1.6, 1.7, 2.1

Section 1.6

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$$\int \frac{dx}{x(\ln x)^2} = \int \frac{1}{x(\ln(x))^2}\,dx = \int \frac{1}{\cancel{x}\,u^2}\,\cancel{x}\,du = \int \frac{1}{u^2}\,du$$

let $\boxed{u = \ln(x)}$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\boxed{x\,du = dx}$$

$$= \int u^{-2}\,du = -u^{-1} + C = \frac{-1}{u} + C = \boxed{\frac{-1}{\ln(x)} + C}$$

Recall

$$\boxed{\int x^m\,dx = \frac{1}{m+1}x^{m+1} + C \quad \text{for } m \neq -1}$$

331 $\int \frac{1}{x\ln(x)\ln(\ln(x))}dx = \int \frac{1}{\cancel{x}\, u \ln(u)}\cancel{x}\,du = \int \frac{1}{u\ln(u)}\,du = \int \frac{1}{\cancel{u}\, v}\cancel{u}\,dv$

let $u=\ln(x)$

$\frac{du}{dx}=\frac{1}{x}$

$x\,du = dx$

$v=\ln(u)$

$\frac{dv}{du}=\frac{1}{u}$

$u\,dv = du$

$$= \int \frac{1}{v}dv = \ln|v| + C = \ln|\ln|u|| + C = \boxed{\ln|\ln|\ln|x||| + C}$$

333 $\int \frac{\cos(x)-x\sin(x)}{x\cos(x)}dx = \ln|g(x)| + C = \boxed{\ln|x\cos(x)| + C}$

$\frac{g'(x)}{g(x)}$ or just let $u = x\cos(x)$.

334 $\int \frac{\ln(\sin(x))}{\tan(x)}dx = \int \frac{u}{\cancel{\tan(x)}}\cancel{\tan(x)}\,du = \int u\,du = \frac{1}{2}u^2 + C$

$$\boxed{= \frac{1}{2}(\ln(\sin(x)))^2 + C}$$

let $u = \ln(\sin(x))$

$\frac{du}{dx} = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$

$\tan(x)\,du = dx$

339. $$\int e^{\tan(x)}\sec^2(x)\,dx = \boxed{e^{\tan(x)}+C}$$

$$e^{g(x)}g'(x) = \frac{d}{dx}e^{g(x)}$$ or let $u=\tan(x)$.

355. $$\int_1^2 \frac{1+2x+x^2}{3x+3x^2+x^3}\,dx = \frac{1}{3}\int_1^2 \frac{3(1+2x+x^2)}{3x+3x^2+x^3}\,dx = \frac{1}{3}\int_1^2 \frac{3+6x+3x^2}{3x+3x^2+x^3}\,dx$$

let $u=3x+3x^2+x^3$
or do the following

$$\frac{g'(x)}{g(x)} = \frac{d}{dx}\ln|g(x)|$$

$$= \frac{1}{3}\ln|3x+3x^2+x^3|\Big]_{x=1}^{x=2} = \boxed{\frac{1}{3}\left(\ln(26)-\ln(7)\right) = \frac{1}{3}\ln\left(\frac{26}{7}\right)}$$

356 $$\int_0^{\frac{\pi}{4}} \tan(x)\,dx = \int_0^{\frac{\pi}{4}} \frac{\sin(x)}{\cos(x)}\,dx = -\int_0^{\frac{\pi}{4}} \frac{-\sin(x)}{\cos(x)}\,dx = -\ln|\cos(x)|\Big]_0^{\frac{\pi}{4}}$$

$$\frac{g'(x)}{g(x)}$$

$$= -\left(\ln\left(\frac{\sqrt{2}}{2}\right) - \ln(1)\right)$$

$$\boxed{= -\ln\left(\frac{1}{\sqrt{2}}\right) = \ln(\sqrt{2})}$$

Remember
$k\ln(a) = \ln(a^k)$
$-\ln(a) = \ln(a^{-1}) = \ln\left(\frac{1}{a}\right)$

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$$\int_{\pi/6}^{\pi/2} \csc(x)\,dx = \int_{\pi/6}^{\pi/2} \csc(x)\,\frac{\csc(x)+\cot(x)}{\csc(x)+\cot(x)}\,dx$$

$$= \int_{\pi/6}^{\pi/2} \frac{\csc^2(x)+\csc(x)\cot(x)}{\cot(x)+\csc(x)}\,dx = -\int_{\pi/6}^{\pi/2} \frac{-\csc^2(x)-\csc(x)\cot(x)}{\cot(x)+\csc(x)}\,dx$$

$$\frac{d}{dx}\left(\cot(x)+\csc(x)\right) =$$

$$-\csc^2(x)-\csc(x)\cot(x)$$

$$\frac{g'}{g}$$

$$= -\ln\left|\cot(x)+\csc(x)\right|\Big]_{\pi/6}^{\pi/2} = -\ln|0+1| + \ln|\sqrt{3}+2| = \boxed{\ln|\sqrt{3}+2|}$$

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$$\int e^{2x}\sqrt{1-e^{2x}}\,dx = \int \cancel{e^{2x}}\sqrt{u}\,\frac{du}{-2\cancel{e^{2x}}} = -\frac{1}{2}\int u^{\frac{1}{2}}\,du$$

let $u = 1-e^{2x}$

$$\frac{du}{dx} = -2e^{2x}$$

$$\boxed{\frac{du}{-2e^{2x}} = dx}$$

$$= -\frac{1}{\cancel{2}}\,\frac{\cancel{2}}{3}\,u^{\frac{3}{2}} + C$$

$$= -\frac{1}{3}u^{\frac{3}{2}} + C$$

$$\boxed{= -\frac{1}{3}\left(1-e^{2x}\right)^{\frac{3}{2}} + C}$$