Discussion Problems

Sections 3.2, 3.3

(7) $\int \ln(x)\,dx = x\ln(x) - \int dx = \boxed{x\ln(x) - x + C}$

$u = \ln(x) \quad du = \frac{1}{x}dx$

$dv = dx \quad v = x$

(10) example from class

(11) $\int x\sin(2x)\,dx = -\frac{1}{2}x\cos(2x) + \frac{1}{2}\int \cos(2x)\,dx = -\frac{1}{2}x\cos(2x) + \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C$

$= \boxed{-\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C}$

let $u = x \quad du = dx$

$dv = \sin(2x) \quad v = -\frac{1}{2}\cos(2x)$

(21) $\int xe^{-x^2}dx = \int \cancel{x}e^u \frac{du}{\cancel{-2x}} = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + C = \boxed{-\frac{1}{2}e^{-x^2} + C}$

let $u = -x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

This uses integration by parts,

$\int x^3 e^{-x^2}dx = -\frac{1}{2}x^2e^{-x^2} - \int \cancel{2x}\left(-\frac{1}{\cancel{2}}e^{-x^2}\right)dx = -\frac{1}{2}x^2e^{-x^2} + \int xe^{-x^2}dx$

$= \boxed{-\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2} + C}$

let $u = x^2 \quad du = 2x\,dx$

$dv = xe^{-x^2}dx \quad v = -\frac{1}{2}e^{-x^2}$

(25) $\int (\ln(x))^2\,dx = x(\ln(x))^2 - \int \frac{2\ln(x)}{x}\,x\,dx = x(\ln(x))^2 - 2\int \ln(x)\,dx$

$u = (\ln(x))^2 \quad du = \frac{2\ln(x)}{x}\,dx$

$dv = dx \quad v = x$

let $u = \ln(x) \quad du = \frac{1}{x}\,dx$

$dv = dx \quad v = x$

$= x(\ln(x))^2 - 2\left[x\ln(x) - \int dx\right] = \boxed{x(\ln(x))^2 - 2x\ln(x) + 2x + C}$

(30) $\int x\,\text{Arctan}(x)\,dx = \frac{1}{2}x^2\text{Arctan}(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx = \frac{1}{2}x^2\text{Arctan}(x) - \frac{1}{2}\int 1 - \frac{1}{1+x^2}\,dx$

$u = \text{Arctan}(x) \quad du = \frac{1}{1+x^2}\,dx$

$dv = x\,dx \quad v = \frac{1}{2}x^2$

$x^2+1 \overline{)\,x^2}$ (1)

$-(x^2+1)$

-1

$\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$

$= \boxed{\frac{1}{2}x^2\text{Arctan}(x) + \frac{1}{2}\text{Arctan}(x) - \frac{1}{2}x + C}$

(95) $\int_0^{\frac{\pi}{2}} x^2\sin(x)\,dx = -x^2\cos(x) + 2x\sin(x) + 2\cos(x)\Big]_0^{\frac{\pi}{2}} = 0 + \pi + 0 - (0 + 0 + 2) = \pi - 2$

$\int x^2\sin(x)\,dx = -x^2\cos(x) - \int (2x)(-\cos(x))\,dx = -x^2\cos(x) + 2\int x\cos(x)\,dx$

$u = x^2 \quad du = 2x$

$dv = \sin(x) \quad v = -\cos(x)$

$u = x \quad du = dx$

$dv = \cos(x) \quad v = \sin(x)$