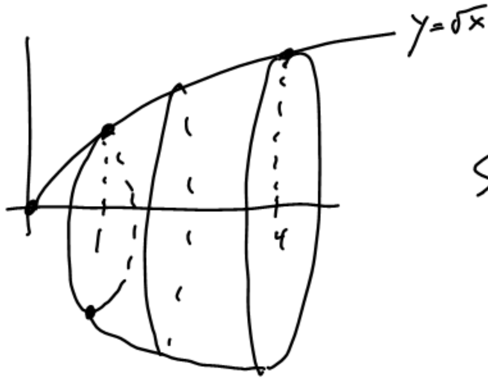
Discussion Problems

Sections 3.1, 3.2, 3.3

Section 2.4



$$\text{Surface area} = \int_1^4 2\pi f(x)\sqrt{1+f'(x)^2}\,dx$$

$$= \int_1^4 2\pi\sqrt{x}\sqrt{1+\left(\frac{1}{2\sqrt{x}}\right)^2}\,dx$$

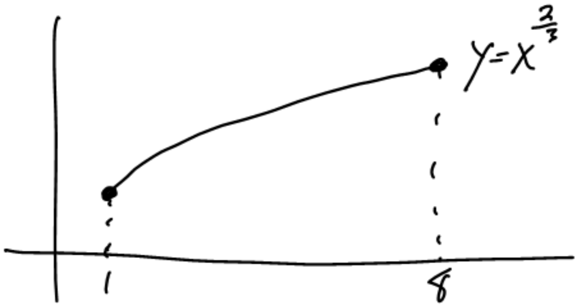
$$= \int_1^4 2\pi\sqrt{x+\tfrac{1}{4}}\,dx = \int_{x=1}^{x=4} 2\pi u^{\frac{1}{2}}\,du = \frac{4}{3}\pi u^{\frac{3}{2}}\Big]_{x=1}^{x=4} = \frac{4}{3}\pi\left(x+\tfrac{1}{4}\right)^{3/2}\Big]_1^4$$

let $u = x+\frac{1}{4}$

$du = dx$

$$\boxed{= \frac{4}{3}\pi\left[\left(\frac{17}{4}\right)^{3/2} - \left(\frac{5}{4}\right)^{3/2}\right]}$$

(172)



$$\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}}$$

$$\text{length} = \int_1^8 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= \int_1^8 \sqrt{1+\tfrac{4}{9}x^{-\frac{2}{3}}}\,dx = \int\sqrt{1+\tfrac{4}{9}u^{-2}}\;3u^2\,du = 3\int u\sqrt{u^2+\tfrac{4}{9}}\,du$$

let $u = x^{\frac{1}{3}}$

$\frac{du}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$

$3x^{\frac{2}{3}}du = dx$

let $v = u^2 + \frac{4}{9}$

$\frac{dv}{du} = 2u$

$\frac{1}{2u}dv = du$

$$= \frac{3}{2}\int \cancel{u}\sqrt{v}\,\frac{1}{\cancel{u}}dv = \frac{3}{2}\int v^{\frac{1}{2}}dv = v^{\frac{3}{2}}\Big]_{x=0}^{x=1}$$

$$= \left(u^2+\frac{4}{9}\right)^{\frac{3}{2}}\Big]_{x=0}^{x=1} = \left(x^{\frac{2}{3}}+\frac{4}{9}\right)^{\frac{3}{2}}\Big]_0^1 = \left(\frac{13}{9}\right)^{3/2} - \left(\frac{4}{9}\right)^{3/2} = \boxed{\frac{1}{27}\left(13^{3/2}-8\right)}$$

(173) $y = \frac{1}{3}(x^2+2)^{\frac{3}{2}}$ $\quad \frac{dy}{dx} = \frac{1}{2}(x^2+2)^{\frac{1}{2}}(2x) = x\sqrt{x^2+2}$

$$\text{length} = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^1 \sqrt{1+x^2(x^2+2)}\,dx = \int_0^1 \sqrt{x^4+2x^2+1}\,dx$$

$$= \int_0^1 \sqrt{(x^2+1)^2}\,dx = \int_0^1 (x^2+1)\,dx = \frac{1}{3}x^3+x\Big]_0^1 = \frac{4}{3}$$

176 $y = \frac{x^3}{3} + \frac{1}{4x}$ $\qquad \frac{dy}{dx} = x^2 - \frac{1}{4x^2}$

for $x=1$ to $x=3$

$$\text{length} = \int_1^3 \sqrt{1 + \left(x^2 - \frac{1}{4x^2}\right)^2}\, dx = \int_1^3 \sqrt{1 + x^4 - \frac{1}{2} + \frac{1}{16x^4}}\, dx$$

$$= \int_1^3 \sqrt{x^4 + \frac{1}{2} + \frac{1}{16x^4}}\, dx = \int_1^3 \sqrt{\left(x^2 + \frac{1}{4x^2}\right)^2}\, dx = \int_1^3 x^2 + \frac{1}{4x^2}\, dx$$

$$= \left.\frac{x^3}{3} - \frac{1}{4x}\right]_1^3 = \left(9 - \frac{1}{12}\right) - \left(\frac{1}{3} - \frac{1}{4}\right) = \cdots$$

Section 3.1

Log
Inverse trig
Polynomial
Exponential
Trig.

9 $\int \text{Arctan}(x)\, dx$

let $u = \text{Arctan}(x)$ $\quad du = \frac{1}{1+x^2}dx$

$dv = dx$ $\quad v = x$

$\frac{g'}{g}$

$$\int \text{Arctan}(x)\, dx = x\,\text{Arctan}(x) - \int \frac{x}{1+x^2}\, dx = x\,\text{Arctan}\,x - \frac{1}{2}\int \frac{2x}{1+x^2}\, dx$$

$$\boxed{= x\,\text{Arctan}(x) - \frac{1}{2}\ln(1+x^2) + C}$$

(7) $$\int \ln(x)\,dx = x\ln(x) - \int dx = \boxed{x\ln(x) - x + C}$$

$u = \ln(x) \quad du = \frac{1}{x}dx$

$dv = dx \quad v = x$

(10) example from class

(11) $$\int x\sin(2x)\,dx = -\frac{1}{2}x\cos(2x) + \frac{1}{2}\int\cos(2x)\,dx = -\frac{1}{2}x\cos(2x) + \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C$$

$$\boxed{= -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C}$$

let $u = x \quad du = dx$

$dv = \sin(2x) \quad v = -\frac{1}{2}\cos(2x)$

(21) $$\int xe^{-x^2}dx = \int \cancel{x}e^{u}\frac{du}{-2\cancel{x}} = -\frac{1}{2}\int e^{u}\,du = -\frac{1}{2}e^{u} + C = \boxed{-\frac{1}{2}e^{-x^2} + C}$$

let $u = -x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

This uses integration by parts

$$\int x^3e^{-x^2}dx = -\frac{1}{2}x^2e^{-x^2} - \int \cancel{2}x\left(-\frac{1}{\cancel{2}}e^{-x^2}\right)dx = -\frac{1}{2}x^2e^{-x^2} + \int xe^{-x^2}dx$$

$$\boxed{= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2} + C}$$

let $u = x^2 \quad du = 2x\,dx$

$dv = xe^{-x^2}dx \quad v = -\frac{1}{2}e^{-x^2}$

25 $$\int (\ln(x))^2 dx = x(\ln(x))^2 - \int \frac{2\ln(x)}{\cancel{x}} \cancel{x}\, dx = x(\ln(x))^2 - 2\int \ln(x)\, dx$$

$u = (\ln(x))^2 \quad du = \frac{2\ln(x)}{x} dx$

$dv = dx \quad v = x$

let $u = \ln(x) \quad du = \frac{1}{x} dx$

$dv = dx \quad v = x$

$$= x(\ln(x))^2 - 2\left[x\ln(x) - \int dx\right] = \boxed{x(\ln(x))^2 - 2x\ln(x) + 2x + C}$$

30 $$\int x \operatorname{Arctan}(x)\, dx = \frac{1}{2}x^2 \operatorname{Arctan}(x) - \frac{1}{2}\int \frac{x^2}{1+x^2} dx = \frac{1}{2}x^2 \operatorname{Arctan}(x) - \frac{1}{2}\int 1 - \frac{1}{1+x^2} dx$$

$u = \operatorname{Arctan}(x) \quad du = \frac{1}{1+x^2} dx$

$dV = x\, dx \quad V = \frac{1}{2}x^2$

$$\begin{array}{r|l} & 1 \\ x^2+1 & x^2 \\ & -(x^2+1) \\ \hline & -1 \end{array}$$

$$\boxed{\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}}$$

$$\boxed{= \frac{1}{2}x^2 \operatorname{Arctan}(x) + \frac{1}{2}\operatorname{Arctan}(x) - \frac{1}{2}x + C}$$

45 $$\int_0^{\frac{\pi}{2}} x^2 \sin(x)\, dx = -x^2\cos(x) + 2x\sin(x) + 2\cos(x) \Big]_0^{\frac{\pi}{2}} = 0 + \pi + 0 - (0 + 0 + 2) = \pi - 2$$

$$\int x^2 \sin(x)\, dx = -x^2\cos(x) - \int (2x)(-\cos(x))\, dx = -x^2\cos(x) + 2\int x\cos(x)\, dx$$

$u = x^2 \quad du = 2x$

$dv = \sin(x) \quad v = -\cos(x)$

$u = x \quad du = dx$

$dv = \cos(x) \quad v = \sin(x)$

$$= -x^2\cos(x) + 2\left[x\sin(x) - \int \sin(x)\,dx\right] = -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C$$