Exam 1 Tomorrow 5/18 covering 1.6, 1.7, 2.1, 2.2, 2.3

Practice Problems

1.6 321, 329, 333, 337, 357

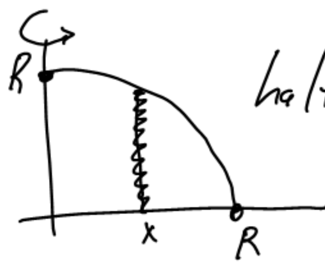
1.7 393, 397, 399

2.1 3, 17, 23

2.2 71, 91, 93, 99

2.3 142, 143, 145, 147

(123) $f(x) = \sqrt{R^2 - x^2} \quad 0 \le x \le R$



half sphere

$$\text{Volume} = \int_0^R 2\pi x\sqrt{R^2 - x^2}\,dx = \int_0^R \cancel{2}\pi \cancel{x}\sqrt{u}\,\frac{du}{\cancel{-2x}}$$

let $u = R^2 - x^2$

$$\frac{du}{dx} = -2x$$

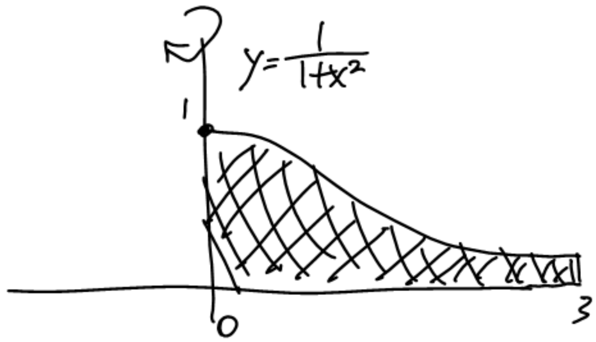
$$\frac{du}{-2x} = dx$$

$$= -\pi\int_{x=0}^{x=R} u^{\frac{1}{2}}\,du = -\frac{2\pi}{3}u^{3/2}\Big]_{x=0}^{x=R} = -\frac{2\pi}{3}\left(R^2 - x^2\right)^{\frac{3}{2}}\Big]_0^R = \frac{-2\pi}{3}\left(0 - (R^2)^{\frac{3}{2}}\right)$$

$$\boxed{= \frac{2\pi}{3}R^3}$$

Volume of The entire sphere $\boxed{\frac{4\pi}{3}R^3}$

(24)



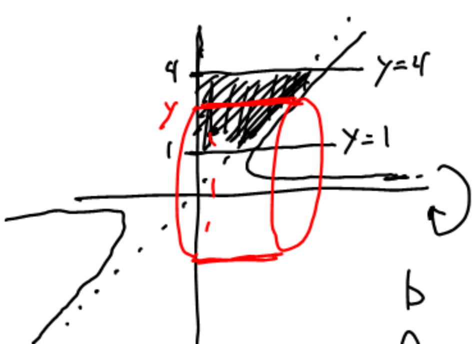
$$Vol = 2\pi\int_0^3 x\,\frac{1}{1+x^2}\,dx$$

$$= \pi\int_0^3 \frac{2x}{1+x^2}\,dx = \pi\ln(x^2+1)\Big]_0^3 = \pi\left(\ln(10)-\ln(1)\right)$$

$$\frac{g'(x)}{g(x)}$$

$$\boxed{=\pi\ln(10)}$$

(34)



$$x = \frac{1+y^2}{y} = \frac{1}{y}+y$$

$$Volume = \int_a^b 2\pi y(\text{height of the shells})\,dy$$

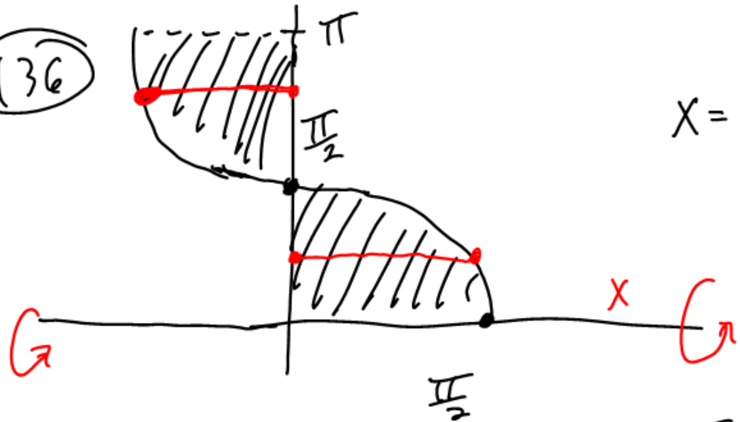
$$= \int_1^4 2\pi \cancel{y}\,\frac{1+y^2}{\cancel{y}}\,dy = 2\pi\int_1^4 1+y^2\,dy$$

$$= 2\pi\left(y+\tfrac{1}{3}y^3\right)\Big]_1^4$$

$$= 2\pi\left(4+\frac{64}{3}-1-\frac{1}{3}\right)$$

$$= 2\pi\left(3+\frac{63}{3}\right) = 48\pi$$

(136)

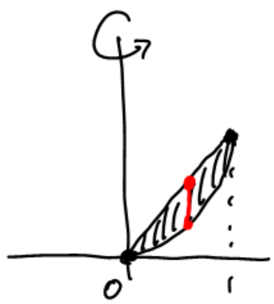


$x = \cos y$

$$\underset{0 \le y \le \frac{\pi}{2}}{\text{Volume}} = 2\pi \int_0^{\frac{\pi}{2}} y\cos(y)\,dy$$

need integration by parts, sec 3.1 to calculate this.

(142)



$$\text{Volume} = 2\pi \int_0^1 x(x - x^2)\,dx$$

$$= 2\pi \int_0^1 x^2 - x^3\,dx = 2\pi\left(\frac{1}{3}x^3 - \frac{1}{4}x^4\right)\Big]_0^1$$

$$= \frac{2\pi}{12} = \frac{\pi}{6}$$

(144)

$y = \frac{1}{4-x}$

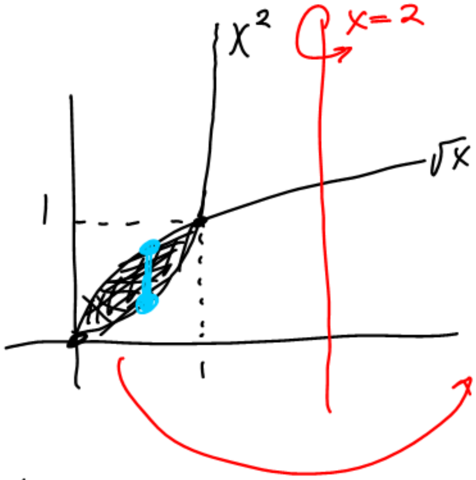
$$\text{Volume} = 2\pi \int_a^b (x - v)(\text{height})\,dx$$

OR

$$2\pi \int_a^b (v - x)(\text{height})\,dx$$

$$\text{Volume} = 2\pi \int_1^2 (4-x)\frac{1}{4-x}\,dx = 2\pi \int_1^2 1\,dx = 2\pi$$

(146)



$$Vol = 2\pi\int_0^1 (2-x)(\sqrt{x}-x^2)\,dx$$

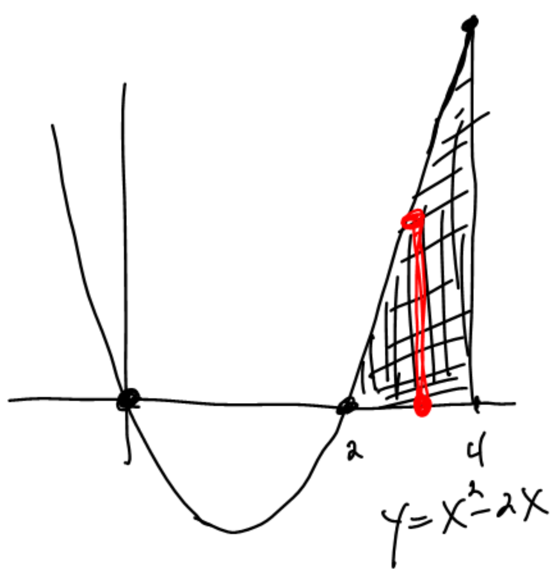
$$= 2\pi\int_0^1 2x^{\frac{1}{2}} - 2x^2 - x^{\frac{3}{2}} + x^3\,dx = 2\pi\left(\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{3}x^3 - \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{4}x^4\right)\Big]_0^1$$

$$= 2\pi\left(\frac{4}{3} - \frac{2}{3} - \frac{2}{5} + \frac{1}{4}\right)$$

$$= 2\pi\ \frac{40-24+15}{60}$$

$$= \frac{31\pi}{30}$$

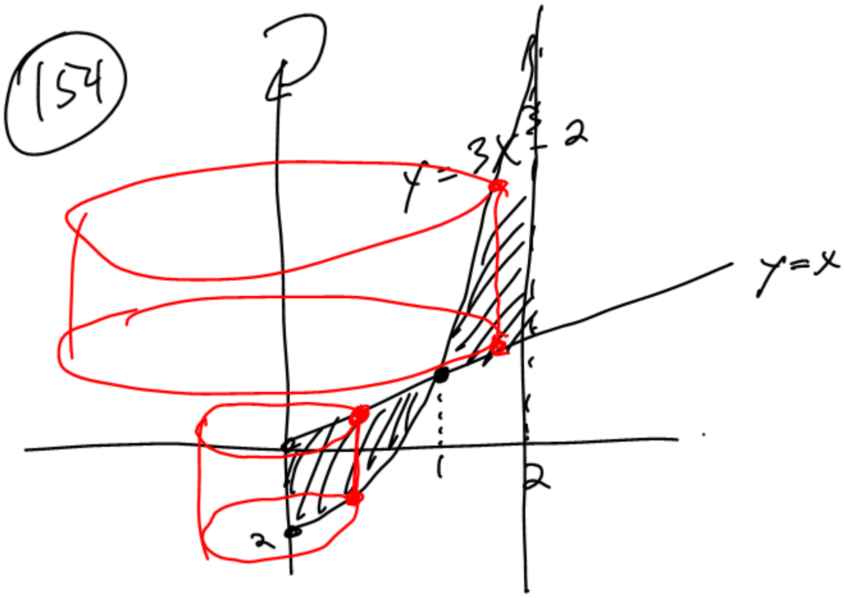
(152)



Volume using shells

$$2\pi\int_2^4 x(x^2-2x)\,dx = \ldots$$

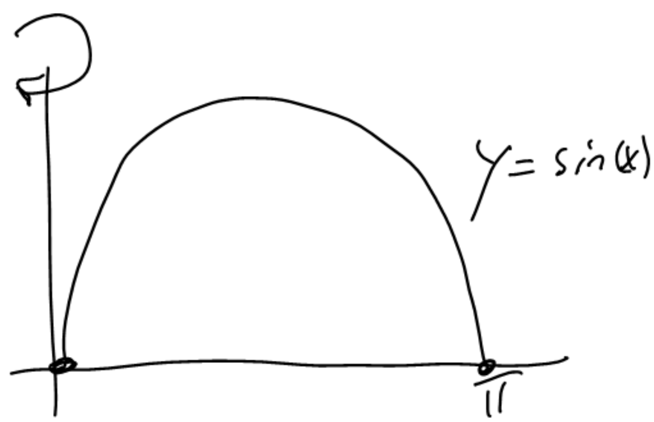
(154)



Rotate around y-axis.

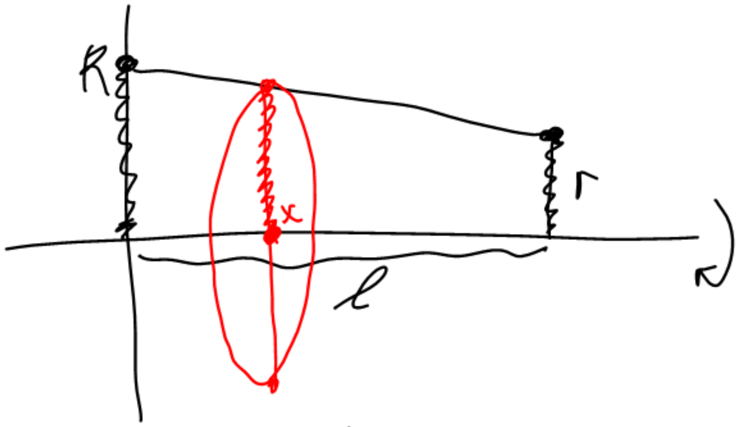
$$\text{Volume} = 2\pi\int_0^1 x\left(x-(3x^3-2)\right)dx + 2\pi\int_1^2 x\left(3x^3-2-x\right)dx$$

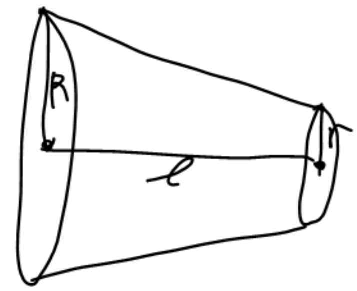
(2.2 | 107)



Volume by Shells

Truncated cone





Calculate the volume.

$$\text{slope} = \frac{r-R}{l}$$

$$\text{line} \quad y = \frac{r-R}{l}x + R$$

$$A(x) = \pi\left(\frac{r-R}{l}x + R\right)^2 = \pi\left[\left(\frac{r-R}{l}\right)^2 x^2 + \frac{2R(r-R)}{l}x + R^2\right]$$

$$\text{Volume} = \int_0^l A(x)\,dx = \pi\int_0^l \left(\frac{r-R}{l}\right)^2 x^2 + \frac{2R(r-R)}{l}x + R^2\,dx$$

$$= \pi\left[\frac{1}{3}\left(\frac{r-R}{l}\right)^2 x^3 + \frac{R(r-R)}{l}x^2 + R^2 x\right]_{x=0}^{x=l}$$

$$= \pi \left[\frac{1}{3} \frac{(r-R)^2}{\cancel{\ell^2}} \ell^{\cancel{3}} + \frac{R(r-R)}{\cancel{\ell}} \ell^{\cancel{2}} + R^2 \ell \right]$$

$$= \pi \ell \left[\frac{(r-R)^2}{3} + R(r-\cancel{R}) + \cancel{R^2} \right]$$

$$= \pi \ell \left[\frac{(r-R)^2}{3} + Rr \right]$$

$$\boxed{= \frac{\pi \ell}{3} \left(R^2 + Rr + r^2 \right)}$$