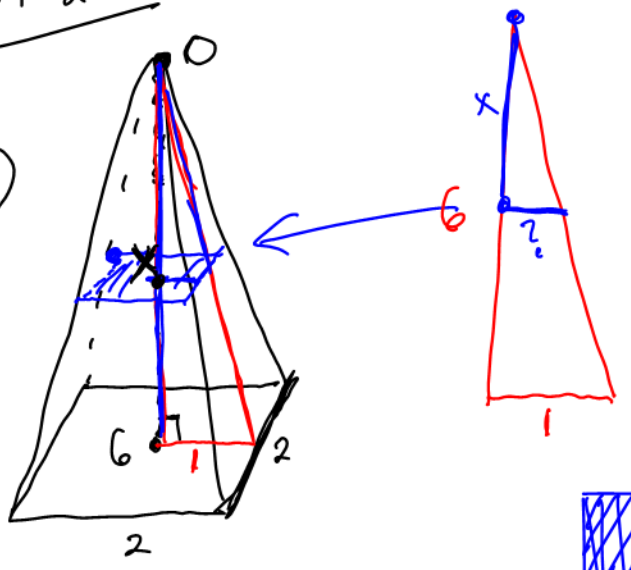
Discussion assignment

Exercises for sections 2.1, 2.2

Exam 1 Thursday May 18 covering 1.6, 1.7 2.1, 2.2, 2.3, 2.4 ??

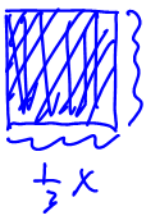
Section 2.2

(63)



$\frac{x}{?} = \frac{6}{1}$

$\frac{1}{6}x = ?$

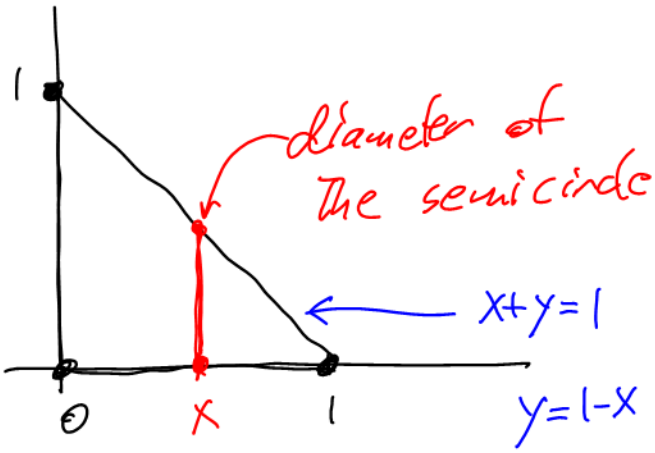


$A(x) = \frac{1}{9}x^2$

$$\text{Volume} = \int_0^6 A(x)\,dx = \int_0^6 \frac{1}{9}x^2\,dx = \frac{1}{27}x^3\Big]_0^6 = \frac{6^3}{27}$$

$$= \frac{2^3 \cdot 3^3}{3^3} = 8$$

69

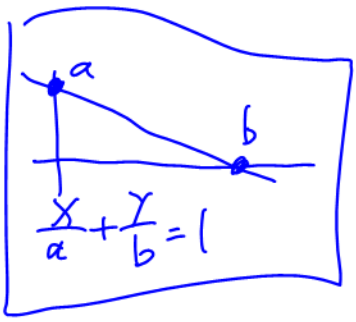


diameter of the semicircle has length $(1-x)$

radius of semicircle $\frac{1}{2}(1-x)$

$$A(x) = \frac{\pi r^2}{2} = \frac{\pi}{2}\cdot\frac{1}{4}(1-x)^2$$

$$= \frac{\pi}{8}(1-x)^2$$



$$\text{Volume} = \int_0^1 \frac{\pi}{8}(1-x)^2\,dx = \frac{-\pi}{8}\int_{0=x}^{1=x} u^2\,du =$$

let $u = 1-x$

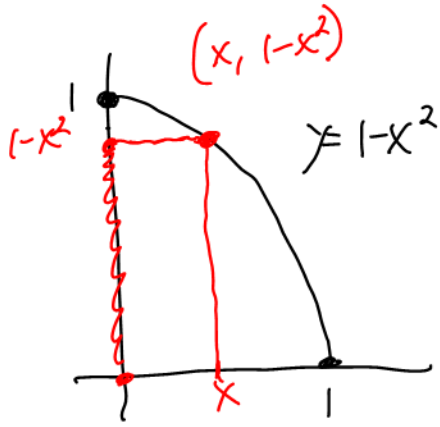
$du = -dx$

$$= \frac{-\pi}{24}u^3\Big]_{x=0}^{x=1} = \frac{-\pi}{24}(1-x)^3\Big]_0^1 = \frac{-\pi}{24}(0-1)$$

$$= \frac{\pi}{24}$$

Try 71, 73 on your own.

(70)



Base of square has length $1-x^2$

$$A(x) = (1-x^2)^2$$

$$\text{Volume} = \int_0^1 (1-x^2)^2 \, dx$$

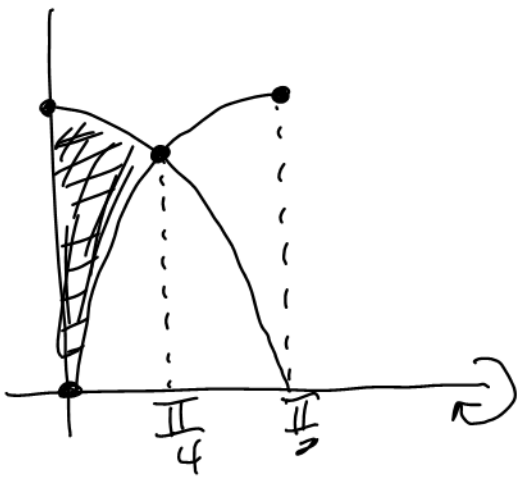
$$= \int_0^1 x^4 - 2x^2 + 1 \, dx$$

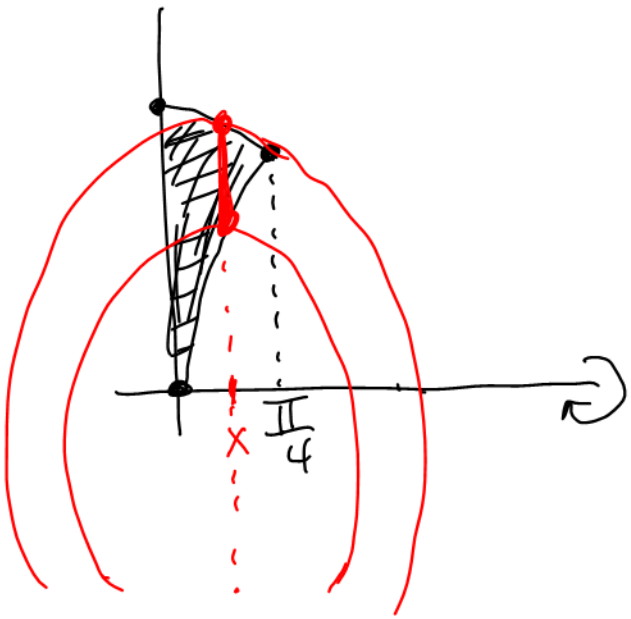
$$= \frac{1}{5}x^5 - \frac{2}{3}x^3 + x \Big]_0^1$$

$$= \frac{1}{5} - \frac{2}{3} + 1$$

$$= \frac{3-10+15}{15} = \frac{8}{15}$$

(79)





$$A(x) = \pi R^2 - \pi r^2$$

$$= \pi\left(\cos^2(x) - \sin^2(x)\right)$$

$$= \pi \cos(2x)$$

$$\text{Volume} = \int_0^{\pi/4} \pi \cos(2x)\,dx = \frac{\pi}{2}\sin(2x)\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{2}\left(\sin\frac{\pi}{2} - \sin(0)\right)$$

let $u = 2x$

$$= \frac{\pi}{2}$$

(1) $x^2 - y^2 = 9 \longrightarrow x^2 - 9 = y^2$

$\sqrt{x^2-9} = |y|$ ← 1st Quadrant

$x + y = 9$

$\sqrt{x^2-9} = y$

x-axis, y-axis

$x^2 - y^2 = 9$

$(x-y)(x+y) = 9$

$x + y = 9$

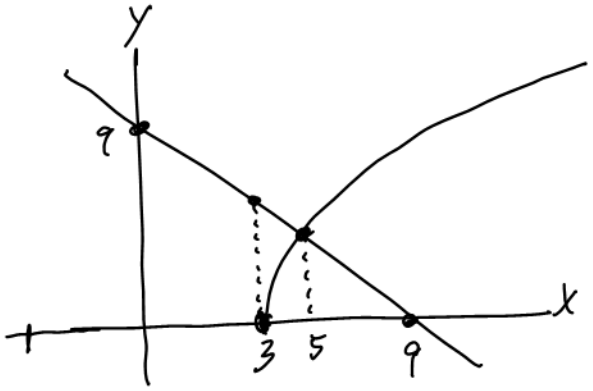
$x - y = 1$

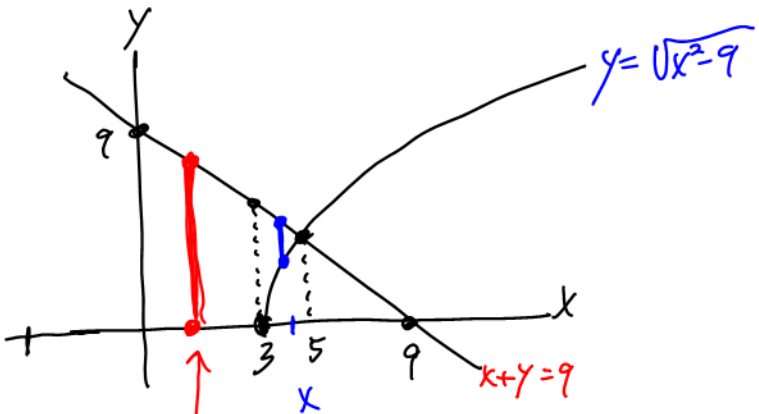
$x + y = 9$

$2x = 10$

$x = 5$

$y = 4$





slice at x is a circle of radius $9-x$

$A(x)=\pi(9-x)^2$

Slice at x has area $A(x)=\pi R^2-\pi r^2$

$=\pi(9-x)^2-\pi(\sqrt{x^2-9})^2$

$$\text{Volume} = \int_0^3 \pi(9-x)^2\,dx + \int_3^5 \pi(9-x)^2-\pi(x^2-9)\,dx =$$

$$= \int_0^3 \pi(9-x)^2\,dx + \int_3^5 \pi(9-x)^2 - \int_3^5 \pi(x^2-9)\,dx$$

$$= \int_0^5 \pi(x-9)^2\,dx - \int_3^5 \pi(x^2-9)\,dx$$

$$= \frac{\pi}{3}(x-9)^3\Big]_0^5 - \pi\left(\frac{1}{3}x^3-9x\right)\Big]_3^5$$

$$= \frac{\pi}{3}(-64+9^3) - \pi\left(\frac{125}{3}-45-(9-27)\right)$$

$$= \pi\left(\frac{729-64}{3} - \left(\frac{125}{3} - 45 - 9 + 27\right)\right)$$

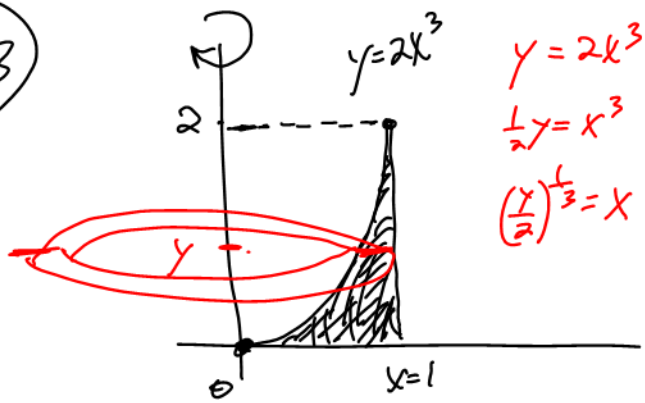
$$\pi\left(\frac{665}{3} - \frac{125}{3} + 27\right)$$

$$\pi\left(\frac{540}{3} + 27\right) = \boxed{207\pi}$$

(77) omit.

Do 83, 85, 89

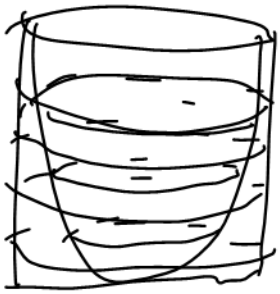
(83)



$y = 2x^3$

$\frac{1}{2}y = x^3$

$\left(\frac{y}{2}\right)^{\frac{1}{3}} = x$



$A(y) = \pi R^2 - \pi r^2$

$= \pi 1^2 - \pi\left(\frac{y}{2}\right)^{\frac{2}{3}}$

$$\text{Volume} = \int_0^2 A(y)\,dy = \pi\int_0^2 1 - \left(\frac{y}{2}\right)^{\frac{2}{3}} dy = \pi\int_0^2 1 - \left(\frac{1}{2}\right)^{\frac{2}{3}} y^{\frac{2}{3}}\,dy = \pi\left(y - \frac{1}{2^{\frac{2}{3}}}\frac{3}{5}y^{\frac{5}{3}}\right)\Big]_0^2$$

$$= \pi\left(2 - \frac{3}{2^{\frac{2}{3}}\cdot 5}2^{\frac{5}{3}}\right) = \pi\left(2 - \frac{2\cdot 3}{5}\right) = \boxed{\frac{4\pi}{5}}$$