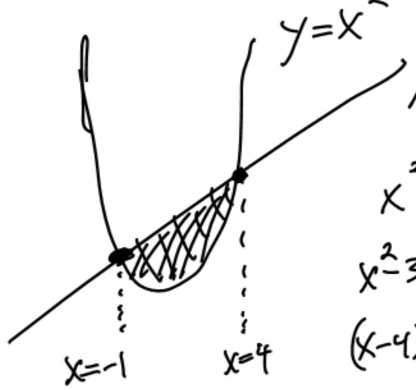
Discussion assignment

Exercises for Sections 2.1, 2.2

Exam 1 Thursday May 18 covering 1.6, 1.7, 2.1, 2.2, 2.3

Section 2.1

(2)



$x^2 = 3x+4$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=-1, 4$

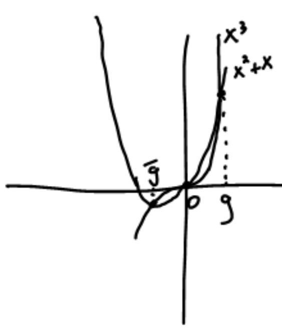
$$\text{Area} = \int_{-1}^{4} (3x+4)-x^2\,dx$$

$$= \frac{3}{2}x^2+4x-\frac{1}{3}x^3\Big]_{-1}^{4}$$

$$= 24+16-\frac{64}{3}-\left(\frac{3}{2}-4+\frac{1}{3}\right)$$

$$= \frac{125}{6}$$

(3)



$x^2+x = x^3$

$0 = x^3-x^2-x$

$0 = x(x^2-x-1)$

$x=0,\ g=\frac{1+\sqrt{5}}{2},\ \bar{g}=\frac{1-\sqrt{5}}{2}$

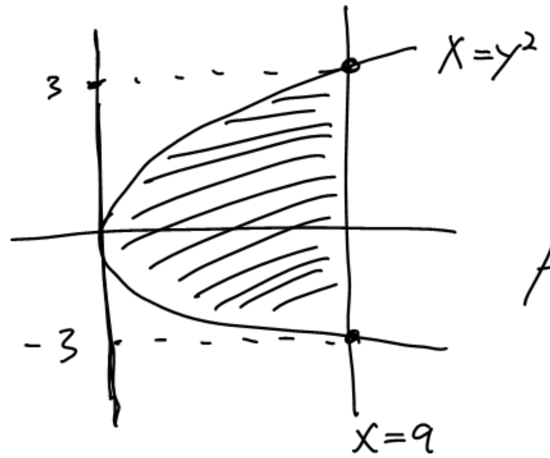
$$\text{Area} = \int_{\bar{g}}^{0} x^3-(x^2+x)\,dx + \int_{0}^{g}(x^2+x)-x^3\,dx$$

$$= \left(\frac{1}{4}x^4-\frac{1}{3}x^3-\frac{1}{2}x^2\right)\Big]_{\bar{g}}^{0} + \left(-\frac{1}{4}x^4+\frac{1}{3}x^3-\frac{1}{2}x^2\right)\Big]_{0}^{g}$$

$$= -\frac{1}{4}\bar{g}^4 + \frac{1}{3}\bar{g}^3 + \frac{1}{2}\bar{g}^2 - \frac{1}{4}g^4 + \frac{1}{3}g^3 - \frac{1}{2}g^2$$

..... leave it here.

(5)



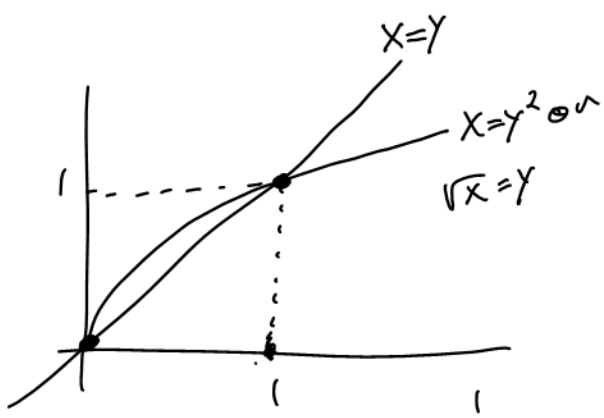
$$\text{Area} = \int_{-3}^{3} 9 - y^2 \, dy$$

$$= 2\int_{0}^{3} 9 - y^2 \, dy$$

$$= 2\left(9y - \frac{1}{3}y^3\right)\Big]_0^3$$

$$= 2(27 - 9) = \boxed{36 \text{ units}^2}$$

(6)



$\sqrt{x} = x$

$x = x^2$

$0 = x^2 - x$

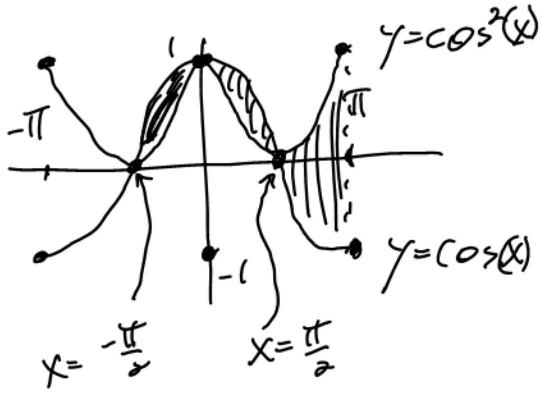
$0 = x(x-1)$

$x = 0, 1$

$$\text{Area} = \int_0^1 (\sqrt{x} - x)\, dx = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\Big]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

$$\text{Area} = \int_0^1 y - y^2 \, dy = \frac{1}{2}y^2 - \frac{1}{3}y^3\Big]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

(9)



$$\text{Area} = 2\int_0^{\frac{\pi}{2}} \cos(x) - \cos^2(x)\,dx$$

$$+2\int_{\frac{\pi}{2}}^{\pi} \cos^2(x) - \cos(x)\,dx$$

$$\boxed{\begin{aligned}\cos^2(\theta) &= \tfrac{1}{2}\left(1+\cos(2\theta)\right)\\ \sin^2(\theta) &= \tfrac{1}{2}\left(1-\cos(2\theta)\right)\end{aligned}}$$

$$\int \cos^2(x)\,dx = \int \frac{1}{2}\left(1+\cos(2x)\right)dx$$

$$= \int \frac{1}{2} + \frac{1}{2}\cos(2x)\,dx$$

$$\boxed{= \frac{1}{2}x + \frac{1}{4}\sin(2x) + C}$$

$$\text{Area} = 2\left(\sin(x) - \frac{1}{2}x - \frac{1}{4}\sin(2x)\right)\Big]_0^{\frac{\pi}{2}}$$

$$+2\left(\frac{1}{2}x + \frac{1}{4}\sin(2x) - \sin(x)\right)\Big]_{\frac{\pi}{2}}^{\pi}$$

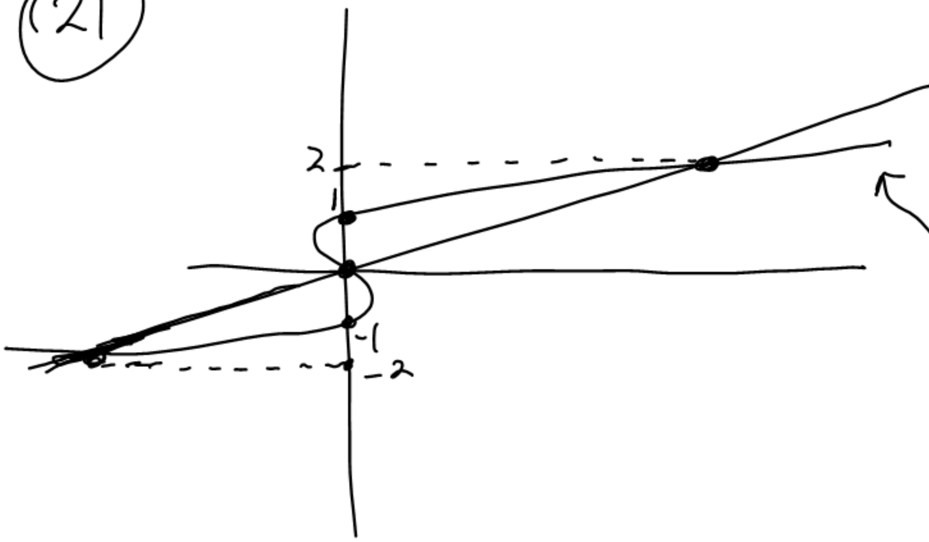
$$= 2\left(1 - \frac{\pi}{4} - 0 - (0-0-0)\right)$$

$$+2\left(\frac{1}{2}\pi + 0 - 0 - \left(\frac{\pi}{4} + 0 - 1\right)\right)$$

$$= 2 - \frac{\pi}{2} + \pi - \frac{\pi}{2} + 2$$

$$= 4$$

(21)



$x = 3y$

$y = \frac{1}{3}x$

$$x = y^3 - y = y(y^2-1)$$
$$= y(y-1)(y+1)$$

$$y^3 - y = 3y$$
$$y^3 - 4y = 0$$
$$y(y^2-4) = y(y-2)(y+2)$$

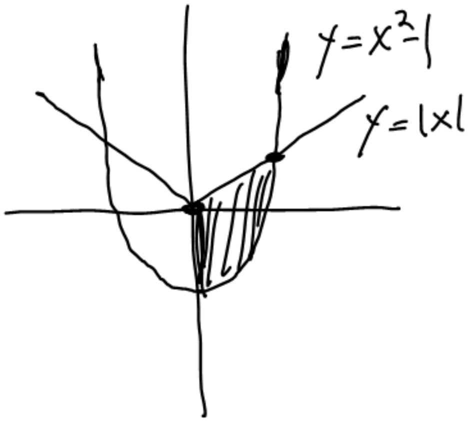
$$\text{Area} = \int_{-2}^{0} (y^3 - y) - 3y \, dy + \int_{0}^{2} 3y - (y^3 - y) \, dy$$

$$= \int_{-2}^{0} y^3 - 4y \, dy + \int_{0}^{2} 4y - y^3 \, dy$$

$$= \frac{1}{4}y^4 - 2y^2 \Big]_{-2}^{0} + 2y^2 - \frac{1}{4}y^4 \Big]_{0}^{2} = 0 - (4-8) + (8-4)$$

$$= 4 + 4 = 8$$

(30)



if $x \geq 0$, Then $|x|=x$.
So to find The intersection point for $x>0$, Solve

$$x^2-1=x$$

$$x^2-x-1=0$$

$$x=g=\frac{1+\sqrt{5}}{2}$$

$$\text{Area}=2\int_0^g x-(x^2-1)\,dx$$

$$=2\left(\frac{1}{2}x^2-\frac{1}{3}x^3+x\right)\Big]_0^g$$

$$=2\left(\frac{1}{2}g^2-\frac{1}{3}g^3+g\right)$$

$$=2\left(\frac{1}{2}(g+1)-\frac{1}{3}(2g+1)+g\right)$$

$$=\frac{13}{3}g+\frac{1}{3}$$