Discussion assignment

Exercises for sections 1.7, 2.1, 2.2

Exam 1 Thursday May 18 covering 1.6, 1.7
2.1, 2.2, 2.3
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Section 1.7

392 $$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \text{Arcsin}(x)\Big]_{x=-\frac{1}{2}}^{x=\frac{1}{2}} = \text{Arcsin}\left(\tfrac{1}{2}\right) - \text{Arcsin}\left(\tfrac{-1}{2}\right)$$

$$= \frac{\pi}{6} - \left(\frac{-\pi}{6}\right) = \frac{\pi}{3}$$

393 $$\int_{\sqrt{3}}^{1} \frac{1}{1+x^2}\,dx = \text{Arctan}(x)\Big]_{\sqrt{3}}^{1} = \text{Arctan}(1) - \text{Arctan}(\sqrt{3})$$

$$= \frac{\pi}{4} - \frac{\pi}{3} = \frac{-\pi}{12}$$

398

$$\int \frac{1}{\sqrt{1-16x^2}}\,dx = \int \frac{1}{\sqrt{1-(4x)^2}}\,dx = \int \frac{1}{\sqrt{1-u^2}}\,\frac{du}{4}$$

let $u = 4x$

$$\frac{du}{dx} = 4$$

$$\frac{du}{4} = dx$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{4}\text{Arcsin}(u) + C = \boxed{\frac{1}{4}\text{Arcsin}(4x) + C}$$

400 like the example done yesterday $\int \frac{1}{9+4x^2}\,dx$

$$\int \frac{1}{25+16x^2}\,dx = \frac{1}{25}\int \frac{1}{1+\left(\frac{4}{5}x\right)^2}\,dx$$

let $u = \frac{4}{5}x$

Consider also the following example

$$\int \frac{1}{\sqrt{25-16x^2}}\,dx = \int \frac{1}{\sqrt{25\left(1-\frac{16}{25}x^2\right)}}\,dx = \int \frac{1}{5\sqrt{1-\frac{16}{25}x^2}}\,dx =$$

$$\frac{1}{5}\int \frac{1}{\sqrt{1-\left(\frac{4}{5}x\right)^2}}\,dx$$

let $u=\frac{4}{5}x$

402

$$\int \frac{1}{|x|\sqrt{4x^2-16}}\,dx = \int \frac{1}{|x|4\sqrt{\frac{x^2}{4}-1}}\,dx$$

$$=\frac{1}{4}\int \frac{1}{|x|\sqrt{\left(\frac{x}{2}\right)^2-1}}\,dx = \frac{1}{4}\int \frac{1}{|\cancel{2}u|\sqrt{u^2-1}}\,\cancel{2}\,du$$

let $u=\frac{x}{2} \longrightarrow 2u=x$

$\frac{du}{dx}=\frac{1}{2}$

$2\,du=dx$

$$=\frac{1}{4}\int \frac{1}{|u|\sqrt{u^2-1}}\,du = \frac{1}{4}\text{Arcsec}(u)+C = \boxed{\frac{1}{4}\text{Arcsec}\left(\frac{x}{2}\right)+C}$$

412 $$\int \frac{1}{\text{Arcsin}(t)\sqrt{1-t^2}}\,dt = \int \frac{1}{u\cancel{\sqrt{1-t^2}}}\cancel{\sqrt{1-t^2}}\,du$$

let $u = \text{Arcsin}(t)$

$$\frac{du}{dt} = \frac{1}{\sqrt{1-t^2}}$$

$$\sqrt{1-t^2}\,du = dt$$

$$= \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|\text{Arcsin}(t)| + C}$$

Remember

$$(a^b)^c = a^{bc}$$

424 $$\int \frac{e^t}{1+e^{2t}}\,dt = \int \frac{e^t}{1+(e^t)^2}\,dt$$

let $u = e^t$

$$\frac{du}{dt} = e^t$$

$$\frac{du}{e^t} = dt$$

$$= \int \frac{\cancel{e^t}}{1+u^2}\,\frac{du}{\cancel{e^t}} = \int \frac{1}{1+u^2}\,du = \text{Arctan}(u) + C$$

$$\boxed{= \operatorname{Arctan}(e^t) + c}$$

4.26

$$\int \frac{1}{t(1+(\ln(t))^2)}\,dt = \cdots = \boxed{\operatorname{Arctan}(\ln(t)) + c}$$

let $u = \ln(t)$