3.2 The derivative as a function of x.

If we calculate $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ There is no need to actually specify a.

After obtaining $f'(a)$ you can change a back to x to get a function $f'(x)$.

Here is an equivalent definition That calculates $f'(x)$ directly.

Def
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Essentially we take The previous definition and let $h = x - a$.

example Let $f(x) = x^3$. Calculate $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2$$

$$= 3x^2 + 0 + 0$$

$$= 3x^2$$

So $f(x) = x^3$ yields $f'(x) = 3x^2$.

Now that we have $f'(x)$ we can calculate $f'(a)$ for as many different values of a that we'd like.

<u>example</u>

If $f(x) = x^3$, Then $f'(x) = 3x^2$

so $f'(1) = 3$, $f'(2) = 12$, $f'(0) = 0$, ... etc.

Question find the equation of the line tangent to

$y = x^3$ which has slope $= \frac{3}{4}$.

Since $f'(x) = 3x^2$ we can find the value

$x = a$ for which $f'(a) = \frac{3}{4}$.

$$\cancel{3}x^2 = \frac{\cancel{3}}{4}$$

$$x^2 = \frac{1}{4}$$

$$x = \frac{1}{2}, \frac{-1}{2}.$$

at $a = \frac{1}{2}$

slope $= \frac{3}{4}$

point of tangency is $\left(\frac{1}{2}, \left(\frac{1}{2}\right)^3\right) = \left(\frac{1}{2}, \frac{1}{8}\right)$

$$\boxed{X - \frac{1}{8} = \frac{3}{4}\left(x - \frac{1}{2}\right)}$$

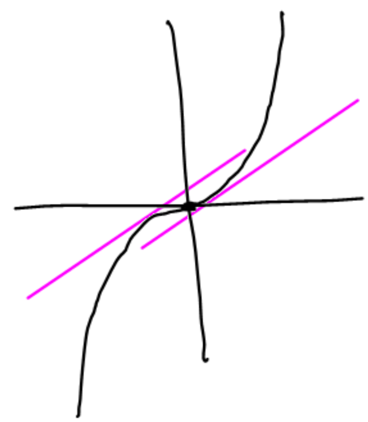
at $a = \frac{-1}{2}$

Slope $= \frac{3}{4}$

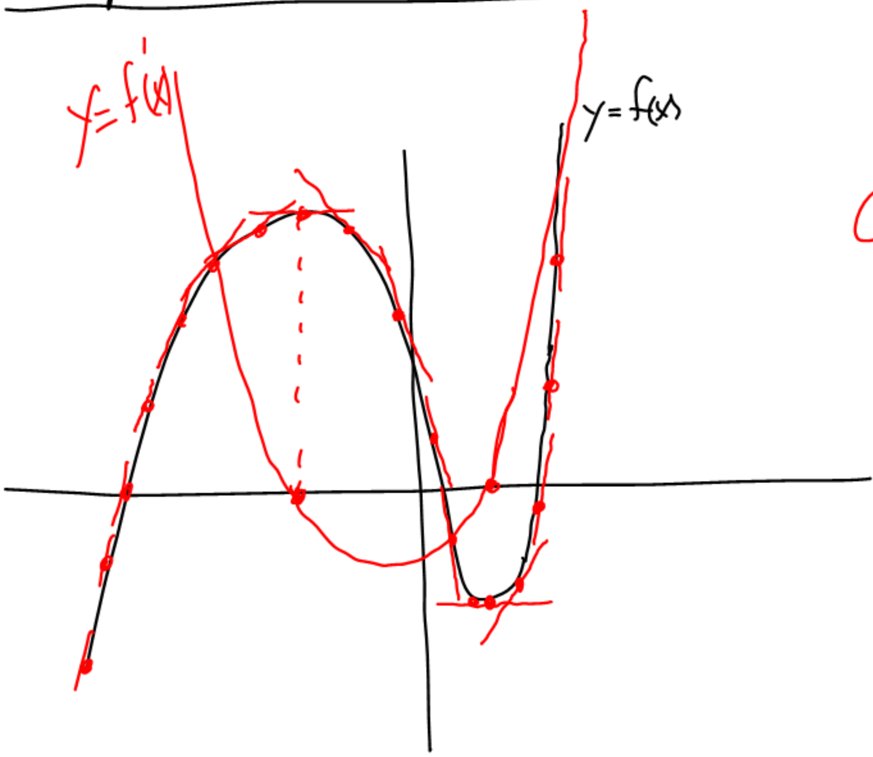
point of tangency is

$\left(\frac{-1}{2}, \frac{-1}{8}\right)$

$$\boxed{y + \frac{1}{8} = \frac{3}{4}\left(x + \frac{1}{2}\right)}$$

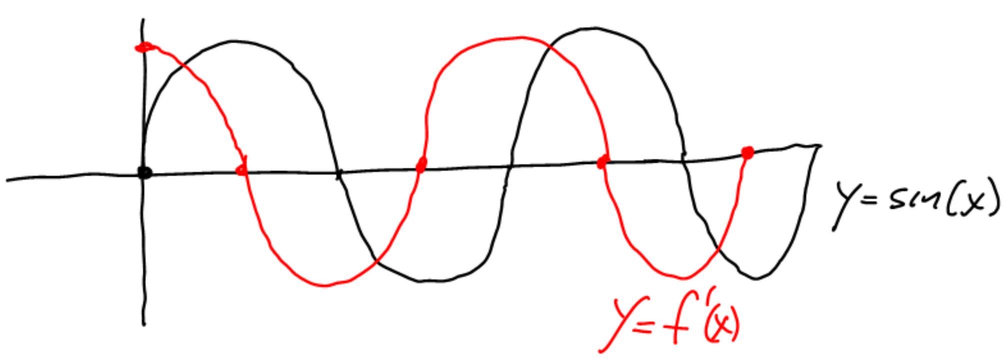


Graphs of f(x) and f'(x) together



Considering the slope of f(x) at at each point the graph y = f(x) suggests a graph y = f'(x) as shown.

How about y = sin(x)



Interpreting the meaning of $f'(x)$ with units

<u>example</u> Suppose that $p(t)$ is the population of some country in number of people at year t.

What is the meaning of $p'(2020)$?

What would it mean if $p'(2020) < 0$?

What would it mean if $p'(2020) > 0$?

$p'(2020)$ is measured in $\frac{\text{people}}{\text{year}}$ and represents the rate of change of population at some instant in the year 2020.

If $p'(2020) > 0$, then population is increasing at the year 2020.

If $p'(2020) < 0$, then population is decreasing at the year 2020.