Section 2.2 The limit of a function.

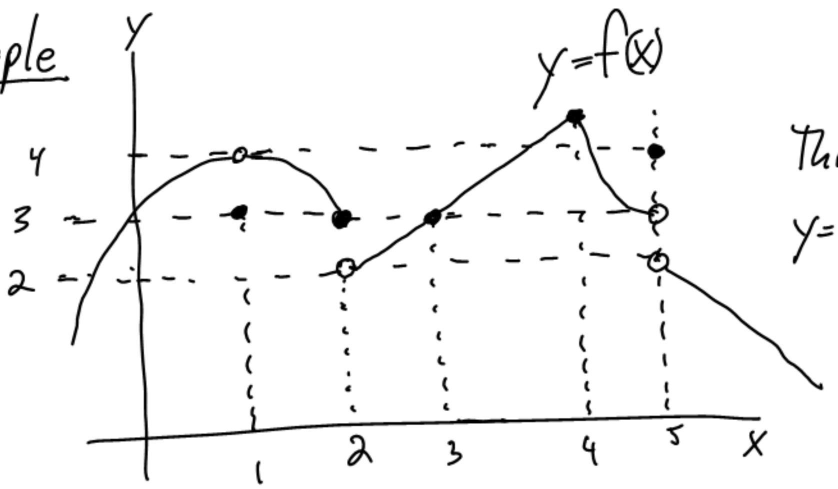
Given a function $f(x)$ we say that

$$\lim_{x \to a} f(x) = L$$

Said as "The limit as x approaches a of $f(x)$ equals L"

when as x gets closer and closer to a but $x \neq a$ then $f(x)$ gets closer and closer to L and does not move away from L.

example



This graph depicts a function $y=f(x)$ for which

$f(1)=3$

$f(2)=3$

$f(3)=3$

$f(5)=4$

$\lim_{x \to 1} f(x) = 4$ but $f(1)=3$

$\lim_{x \to 3} f(x) = 3$ and $f(3)=3$

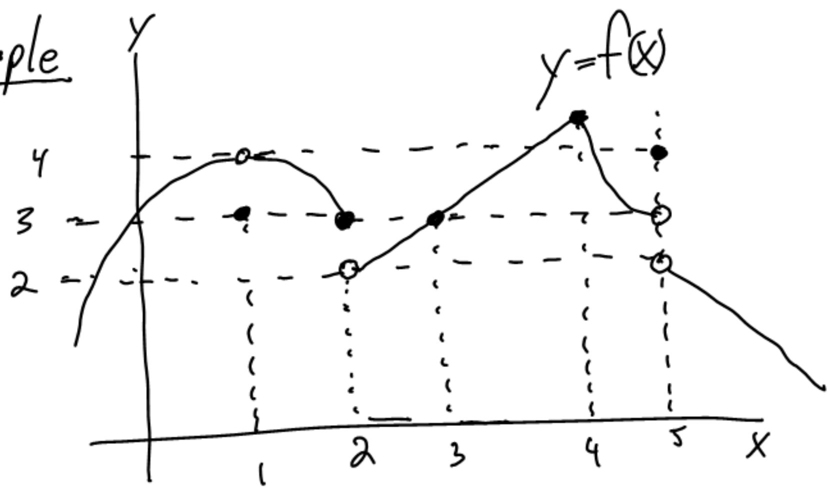
$\lim_{x \to 2}$ does not exist but $f(2)=3$

One-sided limits

$\lim_{x \to a^+} f(x) = L$ As x gets closer and closer to a but $x > a$ then f(x) gets closer and closer to L and does not move away from L.

$\lim_{x \to a^-} f(x) = L$ As x gets closer and closer to a but $x < a$ then f(x) gets closer and closer to L and does not move away from L.

example



$\lim_{x \to 2^-} f(x) = 3$

$\lim_{x \to 2^+} f(x) = 2$

$f(2) = 3$

$\lim_{x \to 5^-} f(x) = 3$

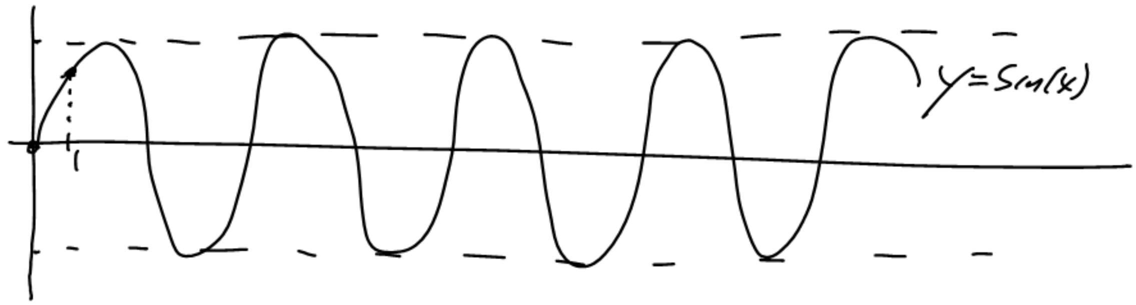
$\lim_{x \to 5^+} f(x) = 2$

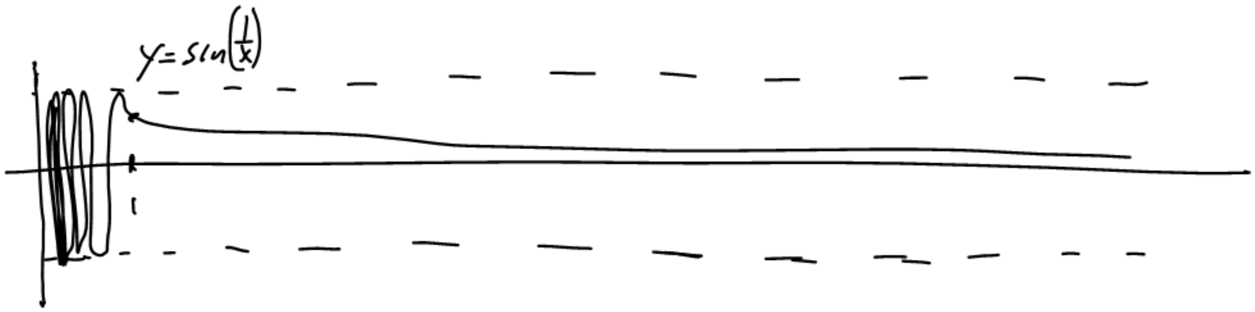
$f(5) = 4$

$\lim_{x \to 2} f(x)$ and $\lim_{x \to 5} f(x)$

both do not exist.

One-sided limits need not always exist, however





$\lim\limits_{x\to 0^+} \sin\left(\frac{1}{x}\right)$ does not exist.

Estimating a limit with calculation tools

Consider $\lim\limits_{x\to 0^+} \frac{\sin(x)}{x}$

x	$\frac{\sin(x)}{x}$
0.1	.9983...
0.01	.99983...
0.001	.999999983...

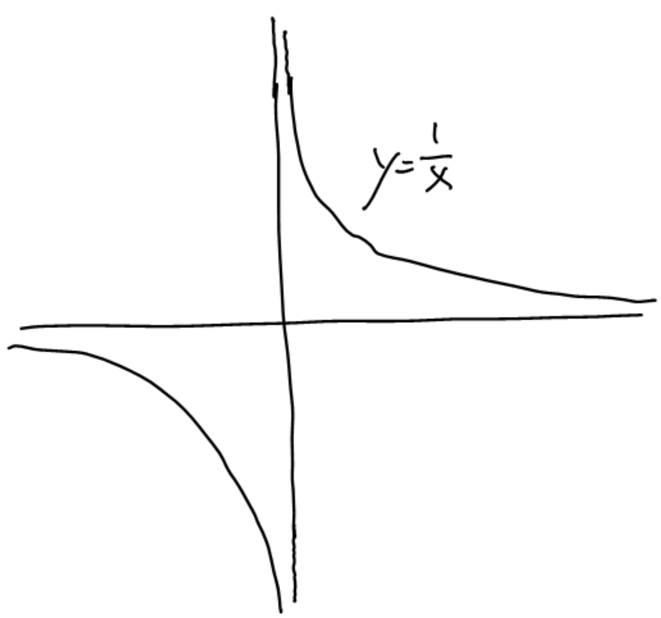
it seems that $\lim\limits_{x\to 0^+} \frac{\sin(x)}{x} = 1$

Infinite limits

* We say that $\lim_{x \to a} f(x) = +\infty$ when as x approaches a, $f(x)$ increases without bound and does not decrease.

* We say that $\lim_{x \to a} f(x) = -\infty$ when as x approaches a, $f(x)$ decreases without bound and does not increase.

These sorts of limits imply a left, right, or double-sided vertical asymptote for $y = f(x)$.



$$\lim_{x \to 0^+} \frac{1}{x} = +\infty$$

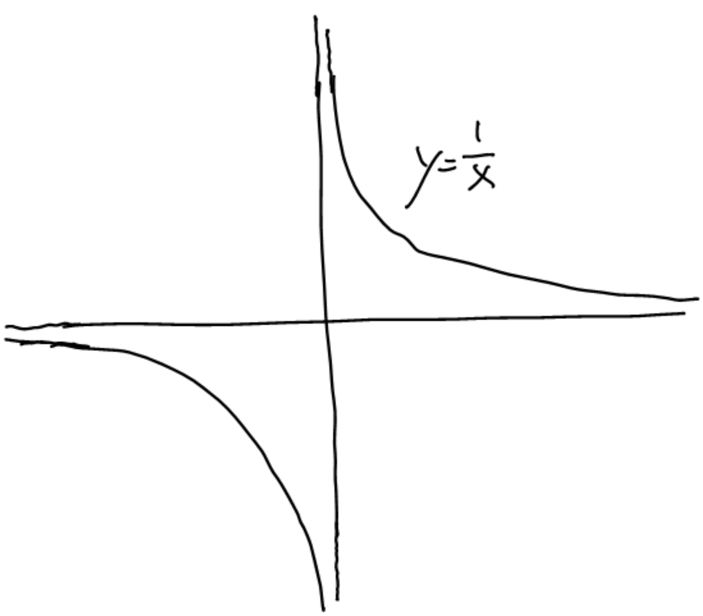
$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$\lim_{x \to 0} \frac{1}{x}$ does not exist.

* We say that $\lim_{x\to+\infty} f(x) = L$ when as x gets larger and larger, $f(x)$ gets closer and closer to L and doesn't move away from L.

* We say that $\lim_{x\to-\infty} f(x) = L$ when as $x<0$ and gets larger and larger, $f(x)$ gets closer and closer to L and doesn't move away from L.

Such limits indicate a horizontal asymptote for $y=f(x)$.



$$\lim_{x\to+\infty} \frac{1}{x} = \lim_{x\to-\infty} \frac{1}{x} = 0$$