Discussion Problems

Section 3.3, 3.4, 3.5

Section 3.3

108 $f(x) = 4x^2 - 7x$

$$f'(x) = 4(2x) - 7(1) = \boxed{8x - 7}$$

110 $f(x) = x^4 + \frac{2}{x} = x^4 + 2x^{-1}$

$$f'(x) = 4x^3 + 2(-x^{-2})$$

$$\boxed{\begin{aligned} &= 4x^3 - 2x^{-2} \\ &= 4x^3 - \frac{2}{x^2} \end{aligned}}$$

115 $f(x) = \dfrac{4x^3 - 2x + 1}{x^2} = x^{-2}(4x^3 - 2x + 1) = 4x - 2x^{-1} + x^{-2}$

Two ways to calculate $f'(x)$

just The power Rule

$$f'(x) = 4 - 2(-x^{-2}) + (-2x^{-3}) = \boxed{4 + 2x^{-2} - 2x^{-3}}$$

using the quotient Rule

$$f'(x) = \frac{(12x^2-2)x^2 - 2x(4x^3-2x+1)}{(x^2)^2}$$

$$= \frac{12x^4 - 2x^2 - 8x^4 + 4x^2 - 2x}{x^4}$$

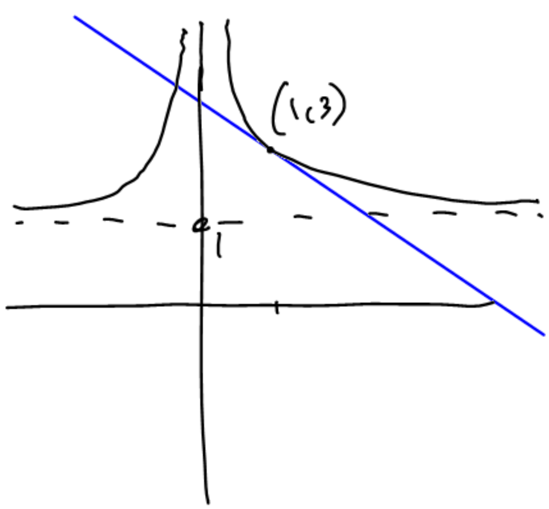
$$= \frac{4x^4 + 2x^2 - 2x}{x^4}$$

$$= x^{-4}(4x^4 + 2x^2 - 2x)$$

$$\boxed{= 4 + 2x^{-2} - 2x^{-3}}$$

Same answer.

(119) $y = \frac{2}{x^2} + 1$ find tangent line at $(1,3)$



$$y = 2x^{-2} + 1$$

$$\frac{dy}{dx} = -4x^{-3}$$

$$\text{slope} = \left.\frac{dy}{dx}\right|_{x=1} = -4(1)^{-3} = -4$$

$$y - b = m(x - a)$$

$$y - 3 = -4(x - 1)$$

$$\boxed{y = -4x + 7}$$

(128) $h(x) = 2x + f(x)g(x)$ find $h'(3)$

$h'(x) = 2 + f'(x)g(x) + f(x)g'(x)$

$h'(3) = 2 + f'(3)g(3) + f(3)g'(3)$

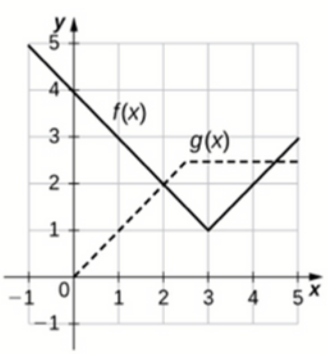
$h'(3) = 2 + (8)(-4) + (-2)(2) = 2 - 32 - 4 = -34$

(126) $h(x) = x f(x) + 4g(x)$ find $h'(1)$

$h'(x) = f(x) + x f'(x) + 4g'(x)$

$h'(1) = f(1) + (1)f'(1) + 4g'(1) = 3 - 1 + 16 = 18$

(132) $h(x) = \dfrac{f(x)}{g(x)}$ find $h'(1)$, $h'(3)$, $h'(4)$



$f'(x) = -1$ for $-1 < x < 3$

$f'(3)$ doesn't exist

$f'(x) = 1$ for $3 < x < 5$

$g'(x) = 1$ for $0 < x < 2.5$

$g'(2.5)$ does not exist

$g'(x) = 0$ for $2.5 < x < 5$

$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$$

$h'(3)$ does not exist because $f'(3)$ doesn't exist.

$$h'(1) = \frac{f'(1)g(1) - g'(1)f(1)}{(g(1))^2} = \frac{(-1)(1) - (1)(3)}{1^2} = -4$$

$$h'(4) = \frac{f'(4)g(4) - g'(4)f(4)}{(g(4))^2} = \frac{(1)(2.5) - 0}{(2.5)^2} = \frac{2}{5}$$

(136) $f(x) = x^3 + x^2 - x - 1$

find the points x at which $f'(x) = 0$

" " " x " " $f'(x) = -1$

$f'(x) = 3x^2 + 2x - 1$

$3x^2 + 2x - 1 = 0$

$(3x-1)(x+1) = 0$

$x = \frac{1}{3}, -1$

$3x^2 + 2x - 1 = -1$

$3x^2 + 2x = 0$

$x(3x+2) = 0$

$x = 0, -\frac{2}{3}$

(144) If t is in units of time

and $f(t)$ is in units of distance

Then $f'(t)$ is in $\frac{\text{units distance}}{\text{unit of time}}$ which is a measure of something called velocity

Also $f''(t)$ is in $\dfrac{\frac{\text{units distance}}{\text{unit time}}}{\text{unit time}} = \frac{\text{units distance}}{(\text{units time})^2}$ which is a rate of change of velocity. This is normally called acceleration

$s(t) = t^3 - 6t^2 + 9t$ meters from some starting point.

(a) when is velocity $= 0$

$v(t) = s'(t) = 3t^2 - 12t + 9$

$$3t^2 - 12t + 9 = 0$$

$$t^2 - 4t + 3 = 0$$

$$(t-3)(t-1) = 0$$

$t = 1$ and 3 seconds

(b)

$$a(t) = s''(t) = v'(t) = 6t - 12$$

$$a(1) = -6 \ \frac{m}{s^2}$$

$$a(3) = 6 \ \frac{m}{s^2}$$