Sections 4.1 and 4.2 Exponential functions.

Let $a > 0$ be a positive real number.

A standard <u>exponential function</u> with <u>base</u> a is of the form $f(x) = K a^x$ for some constant K.

<u>example</u> $f(x) = 6\left(\frac{1}{2}\right)^x$ has base $= \frac{1}{2}$

$$f(0) = 6\left(\frac{1}{2}\right)^0 = 6 \cdot 1 = 6$$

$$f(1) = 6\left(\frac{1}{2}\right)^1 = 6 \cdot \frac{1}{2} = 3$$

$$f(2) = 6\left(\frac{1}{2}\right)^2 = 6 \cdot \frac{1}{4} = \frac{3}{2}$$

$\vdots$ etc.

Common bases used with exponential functions include.

$a = \frac{1}{2}$ for modeling "half-life" problems.

$a = 2$

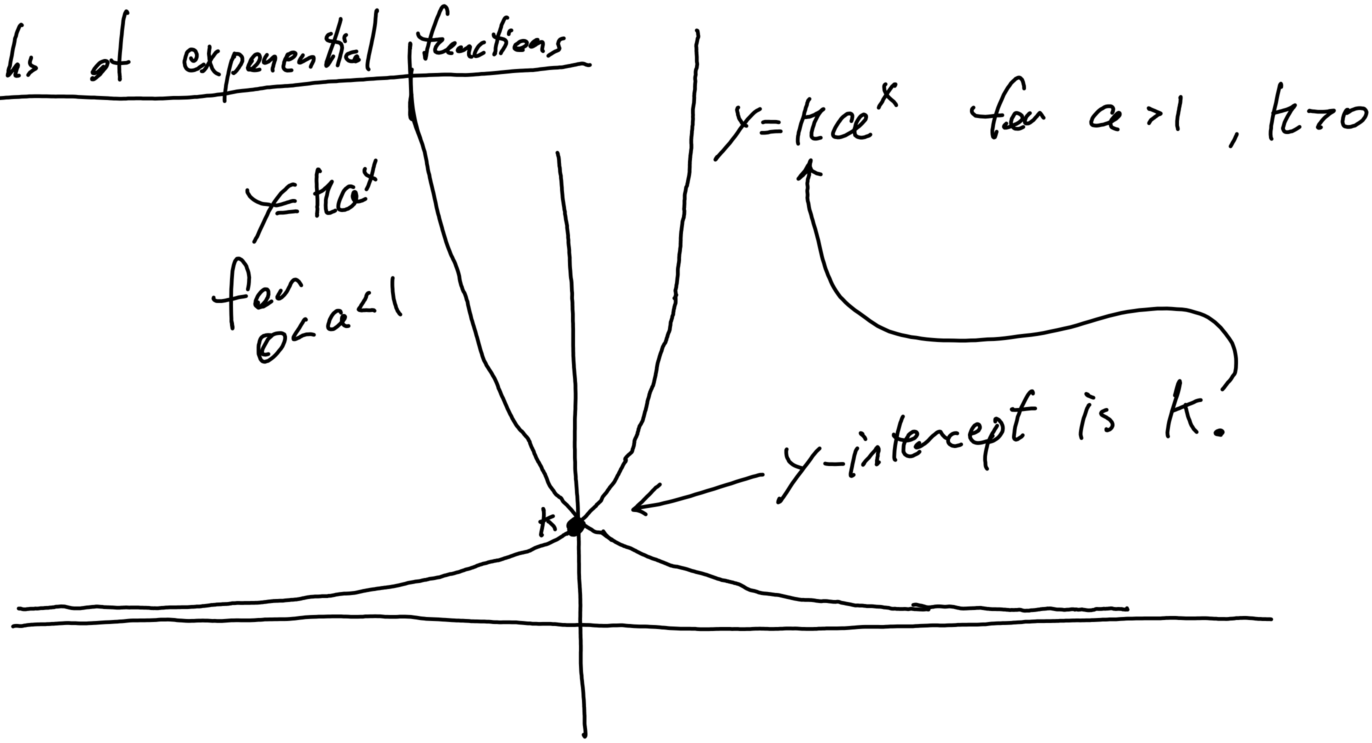
$a = 10$ called the <u>common base</u>

$a = \left(1 + \frac{r}{12}\right)$ where r is an interest rate like $r = .03$ for 3%

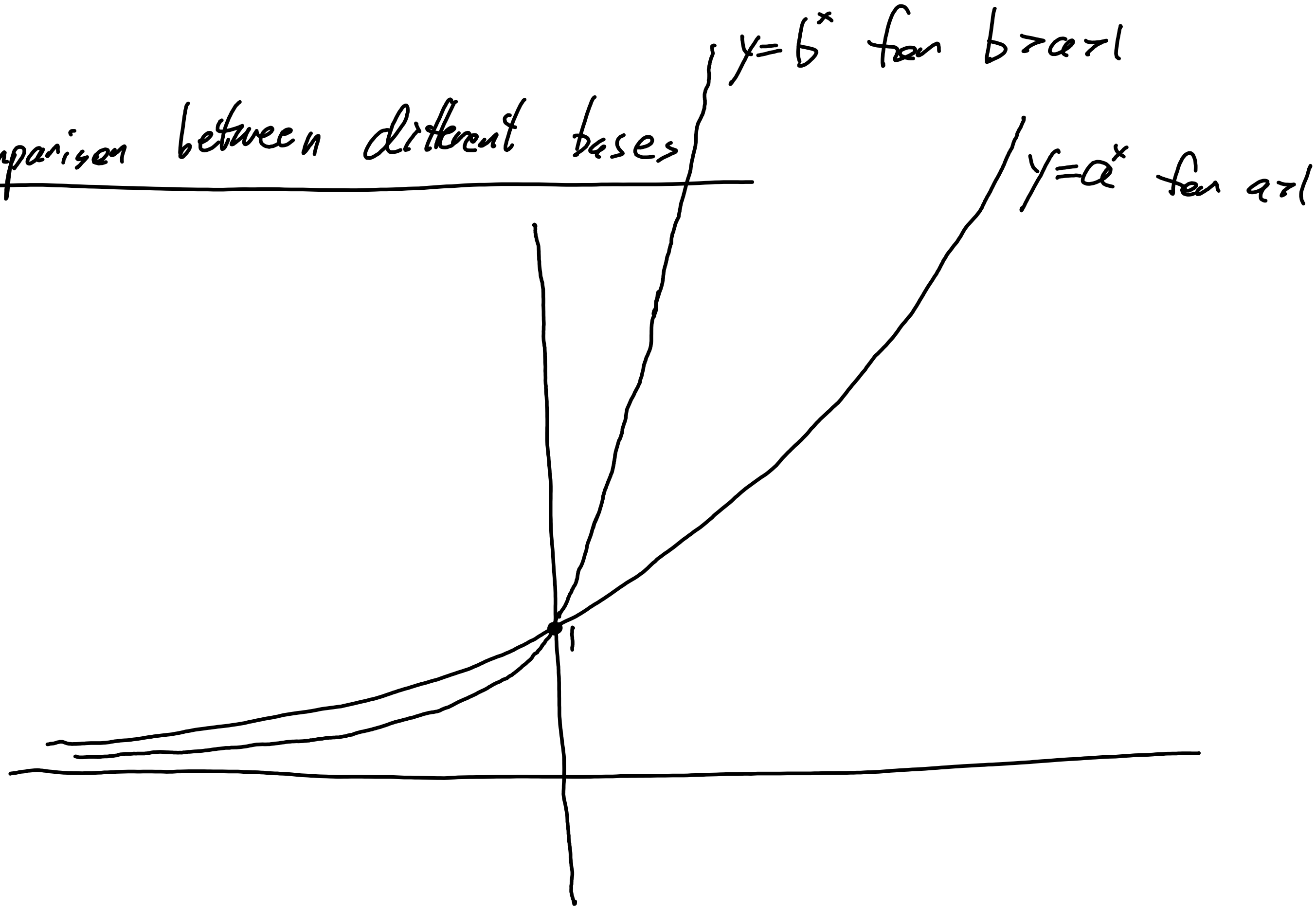
$a = e \approx 2.718\ldots\ldots$ called the natural base

or Euler's Constant.

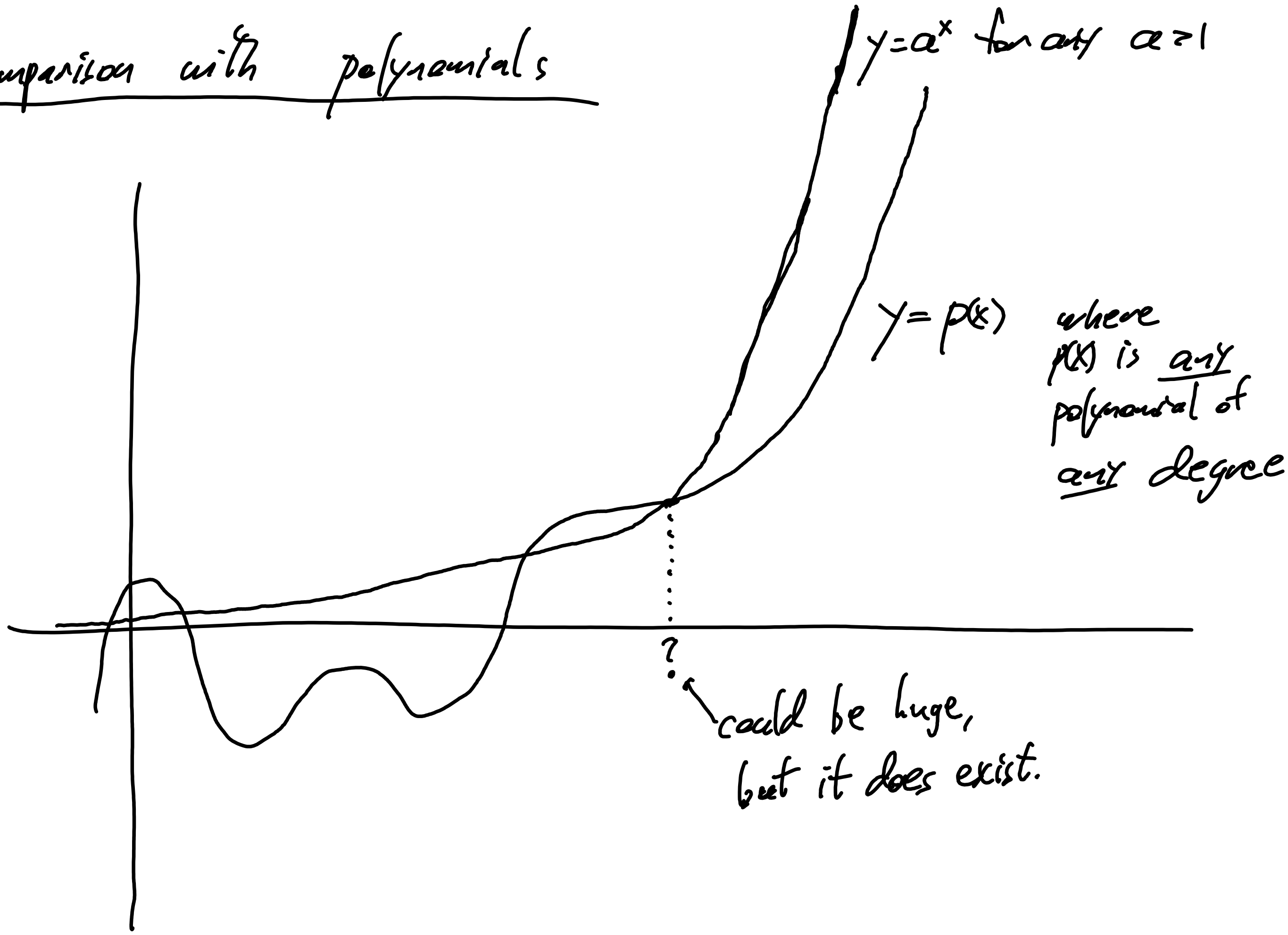
Graphs of exponential functions



Comparison between different bases



Comparison with polynomials



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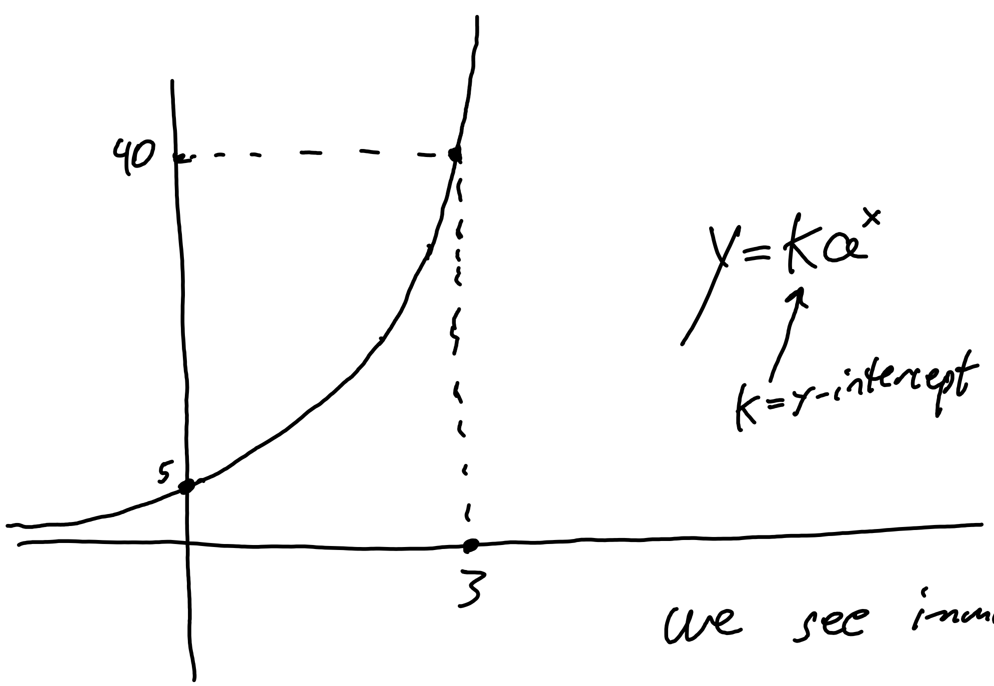
example find the exponential function $y = ka^x$



$y = ka^x$

k = y-intercept

we see immediately that $k = 5$, the y-intercept.

$y = 5a^x$

We also see that $(x,y)=(3,40)$ is a point on the graph. So

$$\frac{40}{5}=\frac{5a^3}{5}$$

$$8=a^3$$

$$\sqrt[3]{8}=a$$

$$2=a$$

So $\boxed{y=5\cdot 2^x}$

<u>example of usage</u>

Carbon-14 is a radioactive isotope of the more common carbon-12. The "half-life" of Carbon-14 is 5730 years. This means that after 5730 years half of a Carbon-14 sample remains of some original sample. So an exponential function which models the remaining amount $A(t)$ of some initial sample I of Carbon-14 after t years is

$$A(t)=I\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

So for example. Take $A(t) = 5\left(\frac{1}{2}\right)^{\frac{t}{5730}}$ mcg.

$A(0) = 5$ mcg

$A(5730) = 5\left(\frac{1}{2}\right)^{\frac{5730}{5730}} = 5\left(\frac{1}{2}\right)^{1} = 2.5$ mcg

$A(10{,}000) = 5\left(\frac{1}{2}\right)^{\frac{10000}{5730}} = 5\left(\frac{1}{2}\right)^{1.745200\ldots} = 1.4914$ mcg

<u>example of usage</u> (Compounding interest)

Suppose that a savings account at a bank pays an interest rate of 2% once per year. This means that an initial investment of I dollars will be worth

$I(1.02)$ after 1-year.

$I(1.02)^2$ after 2-years

$I(1.02)^3$ after 3-years

$\vdots$

$V(t) = I(1.02)^t$ after t years.

e.g. what is the value of $I = 5{,}000$ \$ after 3 years

$V(3) = 5{,}000(1.02)^3 = 5306.04$

example (compounded interest with multiple compounding periods.

If a bank pays an interest rate of r compounded n times a year ($n=4, 12$ is standard) Then The value of an initial investment I after t years is

$$A(t) = I\left(1+\frac{r}{n}\right)^{nt}$$

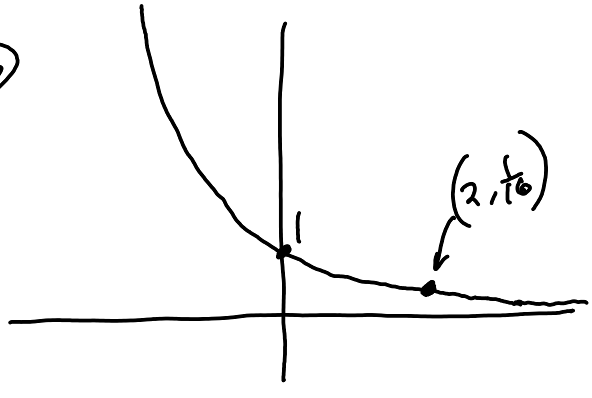
e.g. What is The value of an initial investment of \$5000 after 3 years with 2% interest compounded monthly.

$$A(t) = 5000\left(1+\frac{.02}{12}\right)^{12t}$$

$$A(3) = 5000\left(1+\frac{.02}{12}\right)^{36} = \$5,308.91$$

Section 4.1

(27)



$y = ka^x$ find k and a.

y-intercept $k=1$

$y = a^x$

Then $(x,y) = \left(2, \frac{1}{16}\right)$ is on graph.

$$\sqrt{\frac{1}{16}} = \sqrt{a^2}$$

$$\frac{\sqrt{1}}{\sqrt{16}} = a$$

$$\frac{1}{4} = a \qquad \boxed{y = \left(\frac{1}{4}\right)^x}$$

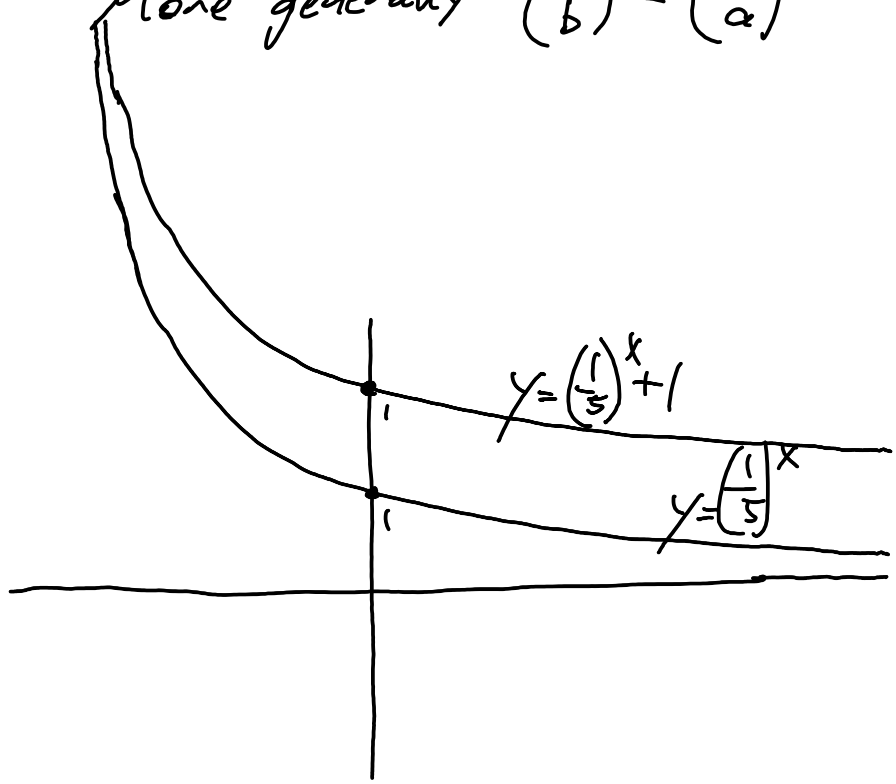
(33) Sketch the graph $y = 5^{-x} + 1$.

Remember $5^{-x} = \frac{1}{5^x}$ More generally $\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$

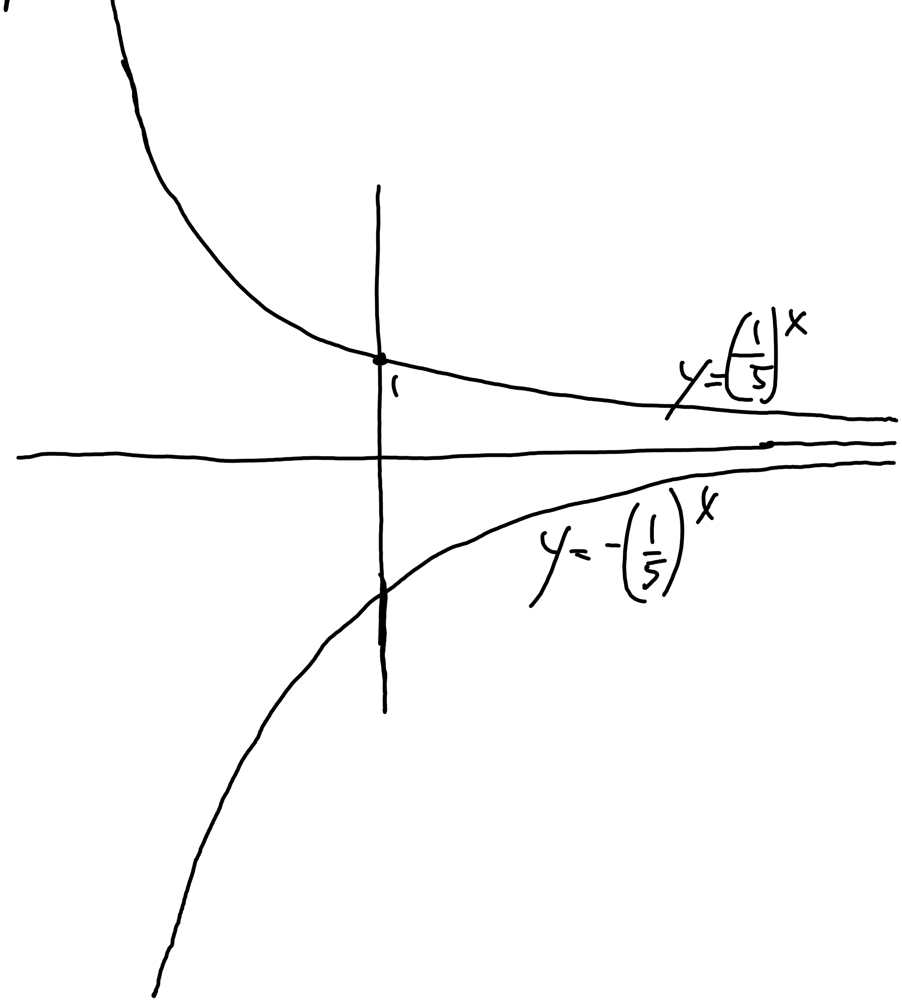
So $y = \frac{1}{5^x} + 1$

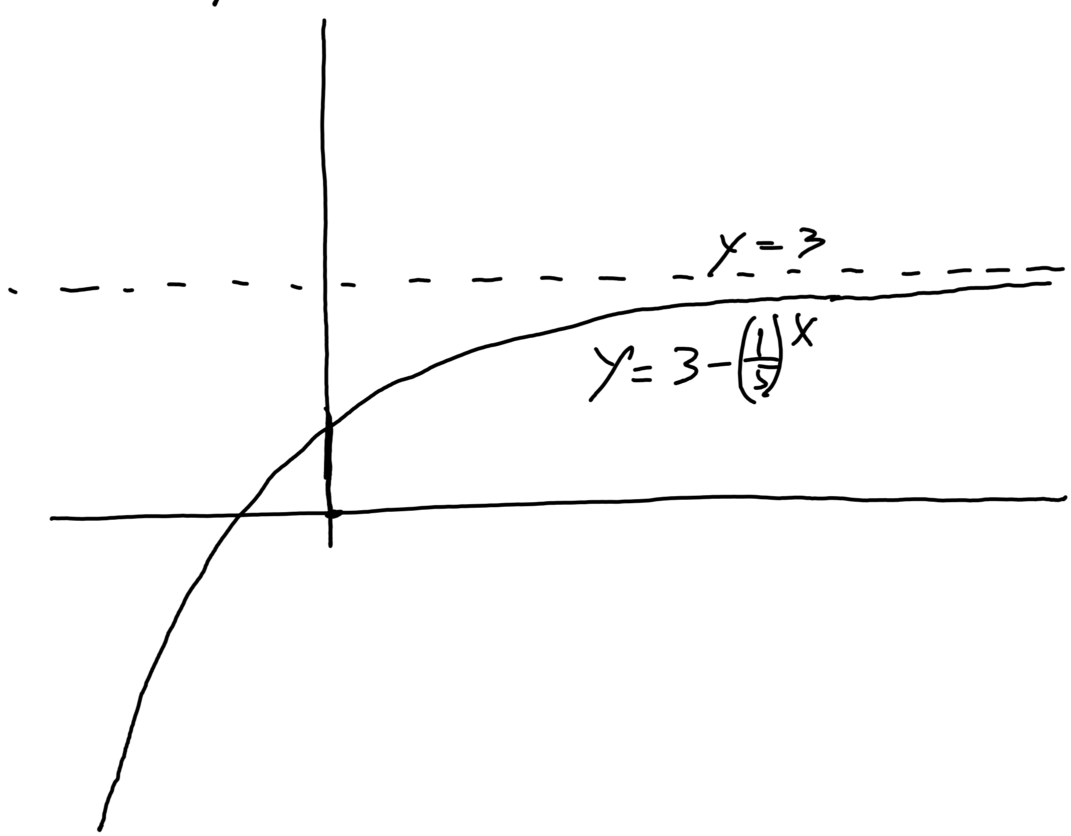
$$y = \frac{1^x}{5^x} + 1$$

$$\boxed{y = \left(\frac{1}{5}\right)^x + 1}$$



(40) Sketch the graph $y = 3 - \left(\frac{1}{5}\right)^x = -\left(\frac{1}{5}\right)^x + 3$





(54) $I = 320$ mice estimates are that the mouse population doubles every year.

(a) Find an exponential function which models mouse population $P(t)$ = number of mice after t years.

$$P(t) = I a^t$$

$$P(t) = 320 \cdot 2^t$$

<u>aside</u> If the mouse population doubles every 2.5 years, then the function is

$$P(t) = 320 \cdot 2^{\frac{t}{2.5}}$$

(59) \$500 is invested at 3.75%/year compounded quarterly.

What is the value of such an investment after 1 year, 2 years, 10 years?

$$V(t) = I\left(1 + \frac{r}{n}\right)^{nt}$$

where t = years

n = compounding periods

$r = \frac{\text{percentage}}{100}$

$$V(t) = 500\left(1 + \frac{.0375}{4}\right)^{4t}$$

$$V(1) = 500\left(1 + \frac{.0375}{4}\right)^{4} = \$519.02$$

$$V(2) = 500\left(1 + \frac{.0375}{4}\right)^{8} = \$538.75$$

$$V(10) = 500\left(1 + \frac{.0375}{4}\right)^{40} = \$726.23$$

4.2

(7) Graph $y = 2 + e^x$