

Section P7

A rational expression is a quotient (or fraction) of two polynomials or just two sums of monomials (Polynomials whose exponents might not be integers).

A rational expression is simplified if either

① The numerator and denominator are fully factored with common factors combined and no negative exponents.

$\frac{P_3 + P_4}{P_3 + P_4}$

OR

② The numerator and denominator are in standard form but without common factors on the top and bottom.

Typically #1 is preferred.

example simplify the following rational expression

$$\frac{x^2 - x - 6}{x^2 - 9} = \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)} = \frac{x+2}{x+3}$$

example $\frac{4(x^2 - 1)}{3 \cdot 2(x^2 + x - 2)} = \frac{x^2 - 1}{3(x^2 + x - 2)} = \frac{(x+1)\cancel{(x-1)}}{3(x+2)\cancel{(x-1)}} = \frac{x+1}{3(x+2)}$

Simplify

example Subtract the two rational expressions to get another rational expression and then simplify it.

$$\frac{1}{x^2 - 1} - \frac{2}{(x+1)(x+2)} = \frac{1}{(x-1)(x+1)} - \frac{2}{(x+1)(x+2)}$$

① factor the denominators

② smallest common denominator
 $(x-1)(x+1)(x+2)$

$$\frac{1}{(x-1)(x+1)} \cdot \frac{x+2}{x+2} - \frac{2}{(x+1)(x+2)} \cdot \frac{x-1}{x-1} =$$

$$\frac{x+2}{(x-1)(x+1)(x+2)} - \frac{2(x-1)}{(x+1)(x+2)(x-1)} =$$

$$\frac{x+2 - 2(x-1)}{(x-1)(x+1)(x+2)} =$$

③ Simplify + factor
the numerator

$$\frac{x+2 - 2x + 2}{(x-1)(x+1)(x+2)} =$$

$$\boxed{\frac{-x + 4}{(x-1)(x+1)(x+2)}}$$

Example Subtract and simplify

$$\frac{x}{x^2+x-2} - \frac{2}{x^2-5x+4} = \frac{x}{(x+2)(x-1)} - \frac{2}{(x-4)(x-1)} =$$

$$\frac{x}{(x+2)(x-1)} \cdot \frac{x-4}{x-4} - \frac{2}{(x-4)(x-1)} \cdot \frac{x+2}{x+2} =$$

$$\frac{x(x-4)}{(x+2)(x-1)(x-4)} - \frac{2(x+2)}{(x-4)(x-1)(x+2)} =$$

$$\frac{x(x-4) - 2(x+2)}{(x+2)(x-1)(x-4)} = \frac{x^2 - 4x - 2x - 4}{(x+2)(x-1)(x-4)} =$$

$$\boxed{\frac{x^2 - 6x - 4}{(x+2)(x-1)(x-4)}}$$

Top doesn't factor.

Nested fractions

Recall that $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \frac{d}{c}$

example resolve the nested fraction into one fraction and simplify.

$$\frac{\frac{x^2+2x+1}{x-2}}{\frac{x+1}{x^2+x-6}} = \frac{x^2+2x+1}{x-2} \cdot \frac{x^2+x-6}{x+1} =$$

$$\frac{(x+1)^2}{x-2} \cdot \frac{(x-2)(x+3)}{x+1} = \frac{(x+1)^{\cancel{2}} \cancel{(x-2)} (x+3)}{\cancel{(x-2)} \cancel{(x+1)}} = \frac{(x+1)(x+3)}{1}$$

$$\boxed{= (x+1)(x+3)}$$

example Simplify the nested fraction into one fraction and then simplify that.

$$\frac{\frac{1}{x} - 1}{1-x} \overset{\substack{= \\ \uparrow \\ \text{subtract} \\ \text{on top}}}{=} \frac{\frac{1}{x} - \frac{x}{x}}{1-x} = \frac{\frac{1-x}{x}}{1-x} =$$

$$\frac{1-x}{x} \cdot \frac{1}{1-x} = \frac{\cancel{1-x}}{x(\cancel{1-x})} = \left(\frac{1}{x} \right)$$