P6 Factoring

A polynomial in standard form is a sum of monomials in decreasing order of exponents.

example $2x^3 - 10x + 2$ Standard form

$6x^{10} - 3x^8 + 2x^7 + 4x$ Standard form.

In factored form the final operation in evaluating a polynomial is multiplication

example $(3x+1)(x-2)(x+4)$

Factoring Polynomials of degree 2

Given x^2+bx+c

IF it can be factored into two degree-1 polynomials $x^2+bx+c=(x+r)(x+s)$

Then we must have that $r+s=b$ and $rs=c$

where b, c, r, s could be positive or negative.

examples

① $x^2+6x+8=(x+2)(x+4)$ because $2+4=6$ and $2\cdot 4=8$

Check $(x+2)(x+4)=x^2+2x+4x+8$

$=x^2+6x+8$ ✓

② $x^2-6x+8=(x-2)(x-4)$ because $-2-4=-6$ $(-2)(-4)=8$

③ $x^2+2x-8=(x-2)(x+4)$ because $-2+4=2$

$(-2)(4)=-8$

④ $x^2-7x-8=(x+1)(x-8)$ because $+1-8=-7$

$(+1)(-8)=-8$

⑤ $x^2-6x-8 \neq (x+a)(x+b)$ for any integers a and b because

$ab=-8$ if and only if

a,b are $\pm 2, \pm 4$ or $\pm 1, \pm 8$

and these never add to -6.

⑥ $x^2-9=(x+3)(x-3)$

$0 \cdot x$

General Form	difference of two squares
$(a+b)(a-b)=a^2-b^2$	

also

$+3-3=0$

$(+3)(-3)=-9$

(7) $6x^2 - 4x = 2x(3x-2)$

More general degree-2 polynomials.

$ax^2+bx+c = (nx+r)(mx+s)$ when $nm = a$

$rs = c$

$ns + rm = b$

examples

$2x^2 - x - 1 = (2x+1)(x-1)$

$2x^2 + x - 1 = (2x-1)(x+1)$

$2x^2 + 3x - 1 = (2x+1)(x-1)$

doesn't factor with integers

Other factoring techniques

A common factor within all terms

example

$$2x^3 - 2x^2 - 12x = \quad * \text{pull out the common factor}$$

$$2x(x^2 - x - 6) = \quad * \text{factor more, if possible.}$$

$$\boxed{2x(x-3)(x+2)}$$

example Integer exponents aren't always necessary with the "common factor" situation

$$2x^{\frac{5}{2}} - 2x^{\frac{3}{2}} - 12x^{\frac{1}{2}} =$$

$$2x^{\frac{1}{2}}(x^2 - x - 6) =$$

$$\boxed{2x^{\frac{1}{2}}(x-3)(x+2)}$$

example $2x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - 12x^{-\frac{1}{2}} =$

$$2x^{-\frac{1}{2}}(x^2 - x - 6) =$$

$$\boxed{2x^{-\frac{1}{2}}(x-3)(x+2)}$$

Grouping

Sometimes polynomials with 4 terms can be factored when grouping the first two and last two terms together as follows

example $5x^3+x^2+5x+1 =$

$(5x^3+x^2)+(5x+1) =$

$x^2(5x+1)+(5x+1) =$

if at this point, there is a common factor of degree 1 then we can factor again.

$\boxed{(5x+1)(x^2+1)}$

factor again if possible

Example

$$9x^3+9x^2-16x-16=$$

$$(9x^3+9x^2)+(-16x-16)=$$

$$9x^2(x+1)-16(x+1)=$$

$$(x+1)(9x^2-16)=$$

$$\boxed{(x+1)(3x-4)(3x+4)}$$

Remember

$a^2-b^2=(a+b)(a-b)$