Section P5 Algebraic Expressions and Polynomials

Given an unknown quantity X (or a "variable")

a monomial in X is an expression like

this

$$rX^{p}$$

p ← usually a rational number.

r ← any real number

examples

$-8X^3$, $17X^{\frac{3}{4}}$, $17X^{\frac{-3}{4}}$, $\sqrt{2}X^4$, $-X^2$, ...

are all monomials.

The number in front of the power of x is called the "coefficient".

If it helps, when the coefficient is a positive integer like $8X^3$ we can think $8X^3 = X^3+X^3+X^3+X^3+X^3+X^3+X^3+X^3$.

One type of algebraic expression in variable x is a <u>sum of monomials</u> using x.

Something like

$$12x^5 - 4x^2 + 5 - 3x^{\frac{1}{2}} + x^{-\frac{5}{2}}$$

A special type of sum of monomials is a <u>polynomial</u> which is a sum of monomials in which all the exponents are integers ≥ 0.

<u>examples of polynomials</u>

$x^3 - 6x^2 + x - 4$	degree 3
$x^{12} - 1$	degree 12
$7x^{10} - 6x^8 + x^6 - x^4 + x^2 - 8$	degree 10
$-6x$	degree 1
-6	degree 0

The <u>degree</u> of the polynomial is the highest power of x.

Something to remember, a number base-10 is a polynomial with $x=10$ and coefficients restricted to $\{0,1,2,...,9\}$.

$385{,}001$ is actually $3(10)^5+8(10)^4+5(10)^3+1(10)^0$

Adding/Subtracting Polynomials

When adding and subtracting, like powers of x are combined by adding the coefficients

e.g.

$$-7x^2+2x^2=-5x^2$$

$$2x^2-7x^2=-5x^2$$

example

$$(6x^4-3x^2+x)+(x^3+7x^2+3x+1)=6x^4+x^3+4x^2+4x+1$$

Multiplying monomials and polynomials

$$(ax^{p})(bx^{q}) = abx^{p+q}$$

example

$$(7x^{3})(-5x^{2}) = -35x^{5}$$

$$\left(\frac{5}{2}t^{4}\right)\left(4t^{\frac{3}{2}}\right) = \left(\frac{5}{2}\right)(4)t^{4+\frac{3}{2}} = 10t^{\frac{11}{2}}$$

When taking the product of two polynomials every monomial in the first is multiplied by every monomial in the second and then they are all added together.

example $(3x^2 - 10x + 5)(x - 2) =$

$$(3x^2)(x) + (3x^2)(-2) + (-10x)(x) + (-10x)(-2) + (5)(x) + (5)(-2) =$$

$$3x^3 - 6x^2 - 10x^2 + 20x + 5x - 10 =$$

$$3x^3 - 16x^2 + 25x - 10$$

Here's another way of organizing a product of polynomials

$$\begin{array}{rrrr}
 & 3x^2 & -10x & +5 \\
 & & x & -2 \\
\hline
 & -6x^2 & -20x & -10 \\
3x^3 & -10x^2 & 5x & 0 \\
\hline
\end{array}$$

$$\boxed{3x^3 - 16x^2 - 25x - 10}$$

example

$$2x^{-\frac{1}{2}}\left(x^2-x+1\right)=\left(2x^{-\frac{1}{2}}\right)\left(x^2\right)+\left(2x^{-\frac{1}{2}}\right)\left(-x\right)+\underline{\left(2x^{-\frac{1}{2}}\right)\left(1\right)}$$

$$\boxed{\begin{aligned}&=2x^{\frac{3}{2}}-2x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\\ &=2x^{\frac{3}{2}}-2x^{\frac{1}{2}}+\frac{2}{x^{\frac{1}{2}}}\end{aligned}}$$

example

$$(a+b)^2=(a+b)(a+b)=a^2+ab+ba+b^2=a^2+ab+ab+b^2$$

$$\boxed{=a^2+2ab+b^2}$$

$$(a+b)^3=(a+b)(a+b)(a+b)=\left(a^2+2ab+b^2\right)(a+b)$$

$$=a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$$

$$\boxed{=a^3+3a^2b+3ab^2+b^3}$$

Pascal's Triangle

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \rightarrow \text{row } 0 \\
 & & & & 1 & & 1 & & & & \rightarrow \text{row } 1 \\
 & & & 1 & & 2 & & 1 & & & \rightarrow \text{row } 2 \\
 & & 1 & & 3 & & 3 & & 1 & & \rightarrow \text{row } 3 \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \rightarrow \text{row } 4 \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 \rightarrow \text{row } 5
\end{array}$$

Binomial Theorem

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$