Section P4 Rational Exponents.

A rational number is a fraction of two integers $\frac{3}{4}$, $\frac{-5}{2}$, $2=\frac{2}{1}$, $0=\frac{0}{1}$, ... etc.

Some non-rational real numbers are $\sqrt{2}$, π, $\frac{1+\sqrt{5}}{2}$ ← golden ratio

Definition

$\sqrt{a}=b$ means that b is a positive real number such that $b^2=a$.

$\sqrt[n]{a}=b$ when n is even means that b is a positive real number such that $b^n=a$.

$\sqrt[n]{a}=b$ when n is odd means that b is a real number such that $b^n=a$.

examples

* $\sqrt[4]{16} = 2$ even though $(-2)^4 = 2^4 = 16$ The fact that 4 is even implies $\sqrt[4]{16}$ is positive.

* $\sqrt{(-3)^2} = \sqrt{9} = 3$

* $\sqrt{x^2} = |x|$

* $\sqrt[3]{-125} = -5$

* $\sqrt{-1}$ is not a "real" number. It's called an "imaginary" number.

* $\sqrt[4]{-1}$ is also "imagmary"!

Definition $a^{\frac{1}{n}} = \sqrt[n]{a}$ e.g. $a^{\frac{1}{2}} = \sqrt{a}$, $a^{\frac{1}{3}} = \sqrt[3]{a}$, ... etc

Now by consequence of this definition

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$$

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} \text{ or } = \sqrt[n]{a^m}$$

Now, all of the same identities from Sec P3 are still true when n is a rational number.

① $(ab)^n = a^n b^n$

② $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

③ $\frac{1}{a^n} = a^{-n}$ and $a^n = \frac{1}{a^{-n}}$

④ $a^m a^n = a^{m+n}$

⑤ $a^n a = a^{n+1}$

⑥ $(a^m)^n = a^{mn}$

⑦ $\frac{a^m}{a^n} = a^{m-n}$

⑧ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Some exercises from the syllabus

Simplify the expressions

59 (a) $X^{\frac{3}{4}} X^{\frac{5}{4}} = X^{\frac{8}{4}} = X^2$

(b) $y^{\frac{2}{3}} y^{\frac{4}{3}} = y^{\frac{6}{3}} = y^2$

61 (a) $\dfrac{w^{\frac{4}{3}} w^{\frac{2}{3}}}{w^{\frac{1}{3}}} = w^{\frac{4}{3}+\frac{2}{3}-\frac{1}{3}} = w^{\frac{5}{3}}$

(b) $\dfrac{a^{\frac{5}{4}}(2a^{\frac{3}{4}})^3}{a^{\frac{1}{4}}} = \dfrac{a^{\frac{5}{4}} 2^3 (a^{\frac{3}{4}})^3}{a^{\frac{1}{4}}} = \dfrac{8 a^{\frac{5}{4}} a^{\frac{9}{4}}}{a^{\frac{1}{4}}} = \dfrac{8 a^{\frac{13}{4}}}{a^{\frac{1}{4}}}$

$= 8a^{\frac{12}{4}} = 8a^3$