Section P3 Integer exponents

Given any real number a and a positive integer n,

$$a^n = \underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_{n \text{ factors}}$$

example

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

$$2^{10} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 1024$$

$$7^1 = 7 \quad \text{and} \quad x^1 = x$$

$$(-3)^3 = (-3) \cdot (-3) \cdot (-3) = -27$$

$$(-2)^4 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 16$$

$$(-8)^1 = -8$$

$$\left(\frac{1}{2}\right)^4 = \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} = \frac{1}{16}$$

Definition $a^0 = 1$ even if, usually, $a = 0$.

Definition $a^{-n} = \dfrac{1}{a^n} = \dfrac{1}{\underbrace{aa\cdots\cdots a}_{n \text{ factors}}} = \underbrace{\dfrac{1}{a}\dfrac{1}{a}\cdots\cdots\dfrac{1}{a}}_{n \text{ factors}}$

Something to watch out for.

The exponent in a^n applies to a only.

So in the expression bca^n this means

$bca^n = bc\underbrace{aa\cdots\cdots a}_{n \text{ factors of } a}$.

In particular

$(-a)^n = \underbrace{(-a)(-a)\cdots\cdots(-a)}_{n \text{ factors}}$

BUT

$-a^n = -\underbrace{aaa\cdots\cdots a}_{n \text{ factors of } a}$.

example

$(-5)^4 = (-5)(-5)(-5)(-5) = 625$

$-5^4 = -5\cdot 5\cdot 5\cdot 5 = -625$

Some arithmetic identities for integer exponents

① $(ab)^n = a^n b^n$ why? $(ab)^n = (ab)(ab)\cdots(ab) = aa\cdots a\,bb\cdots b$
$= a^n b^n$

② $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

③ $\frac{1}{a^n} = a^{-n}$ and $a^n = \frac{1}{a^{-n}}$

④ $a^m a^n = a^{m+n}$ why? $a^m a^n = \underbrace{a\cdots a}_{m \text{ factors}}\ \underbrace{a\cdots a}_{n \text{ factors}} = a^{m+n}$

⑤ $a^n a = a^{n+1}$ why $a^n a = a^n a^1 = a^{n+1}$

⑥ $(a^m)^n = a^{mn}$

⑦ $\frac{a^m}{a^n} = a^{m-n}$ This is a combination of facts 3 and 4.

⑧ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

examples "Simplify" the following expressions by combining common bases and eliminating negative exponents.

① $$\frac{x^4y^2z}{xy^2z^2} = \frac{x^4x^{-1}y^2z}{y^2z^2} = \frac{x^3y^2z}{y^2z^2} = \frac{x^3y^2}{y^2z^2z^{-1}} = \frac{x^3y^2}{y^2z} = \frac{x^3y^2y^{-2}}{z} = \frac{x^3y^0}{z} = \frac{x^3\cdot 1}{z} = \boxed{\frac{x^3}{z}}$$

This way of thinking about the simplification is too slow, however. Here's a quicker method based on the fact $\frac{a^m}{a^n} = a^{m-n}$ also $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$

$$\frac{\cancel{x^4}^{\,2}y^2z}{y^2\cancel{x^2}z^2} = \frac{x^2\cancel{y^2}^{\,0}z}{\cancel{y^2}z^2} = \frac{x^2\cancel{z}}{z^{\cancel{2}\,1}} = \boxed{\frac{x^2}{z}}$$

② $$\frac{\cancel{6}^{\,2\cdot 3}yx^4}{\cancel{3}x^{-2}} = \frac{2yx^4}{\cancel{x^{-2}}} = \frac{2yx^6}{1} = \boxed{2yx^6}$$

(3) $\left(\frac{\cancel{6}^{2} y x^4}{\cancel{3} x^{-2}}\right)^2 = \left(\frac{2 y \cancel{x^4}^{6}}{\cancel{x^{-2}}}\right)^2 = (2yx^6)^2 = 2^2 y^2 (x^6)^2 = 4y^2x^{12}$

One method

another method

$\left(\frac{6yx^4}{3x^{-2}}\right)^2 = \frac{6^2 y^2 (x^4)^2}{3^2 (x^{-2})^2} = \frac{\cancel{36}^{4} y^2 \cancel{x^8}}{\cancel{9} \cancel{x^{-4}}} = 4y^2x^{12}$

Scientific notation for big and small numbers

Consider the powers of 10.

$\vdots$

$10^5 = 100{,}000$

$10^4 = 10{,}000$

$10^3 = 1000$

$10^2 = 100$

$10^1 = 10$

$10^0 = 1$

$10^{-1} = .1$

$10^{-2} = .01$

$10^{-3} = .001$

$10^{-4} = .0001$

$\vdots$

We can use These powers of 10 to write large and small numbers in a compact fashion.

example

$7.35 \times 10^{9} = 7{,}350{,}000{,}000$

This is obtained by moving The decimal dot over to The right 9 places.

or

$7.35 \times 10^{-9} = .00000000735$

This obtained by moving The decimal dot 9 places to The left.