Section 3.6

Graphs of rational functions.

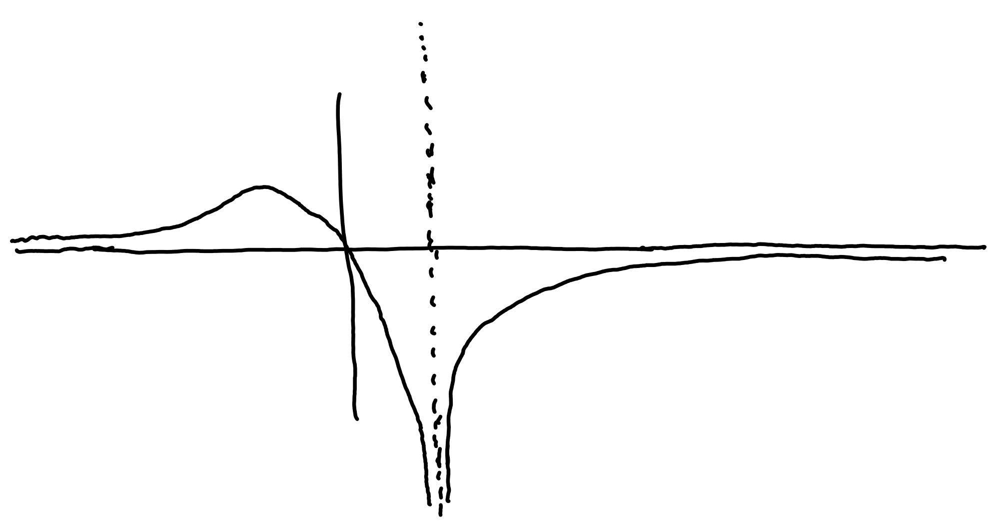
A <u>rational function</u> is $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials.

We will consider what the graph of $y = \frac{p(x)}{q(x)}$ looks like when degree $p(x) \leq$ degree $q(x)$

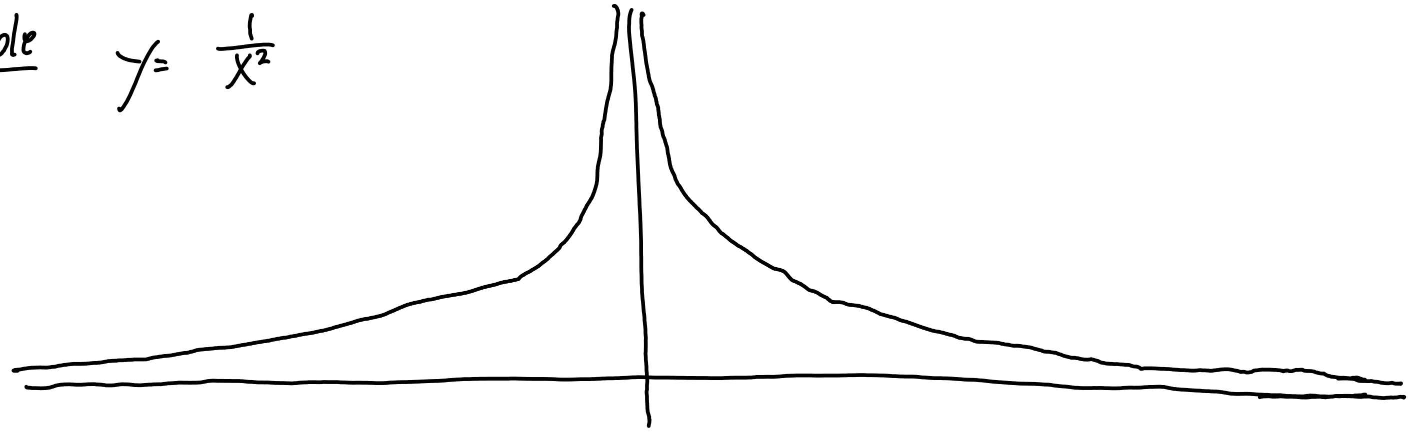
<u>End Behavior</u>

① When degree$(p(x))$ < degree$(q(x))$ Then $y = \frac{p(x)}{q(x)}$ has the line $y=0$ (i.e., the x-axis as a "<u>horizontal asyptote</u>."

<u>example</u> $y = \frac{-x}{(x-1)^2}$

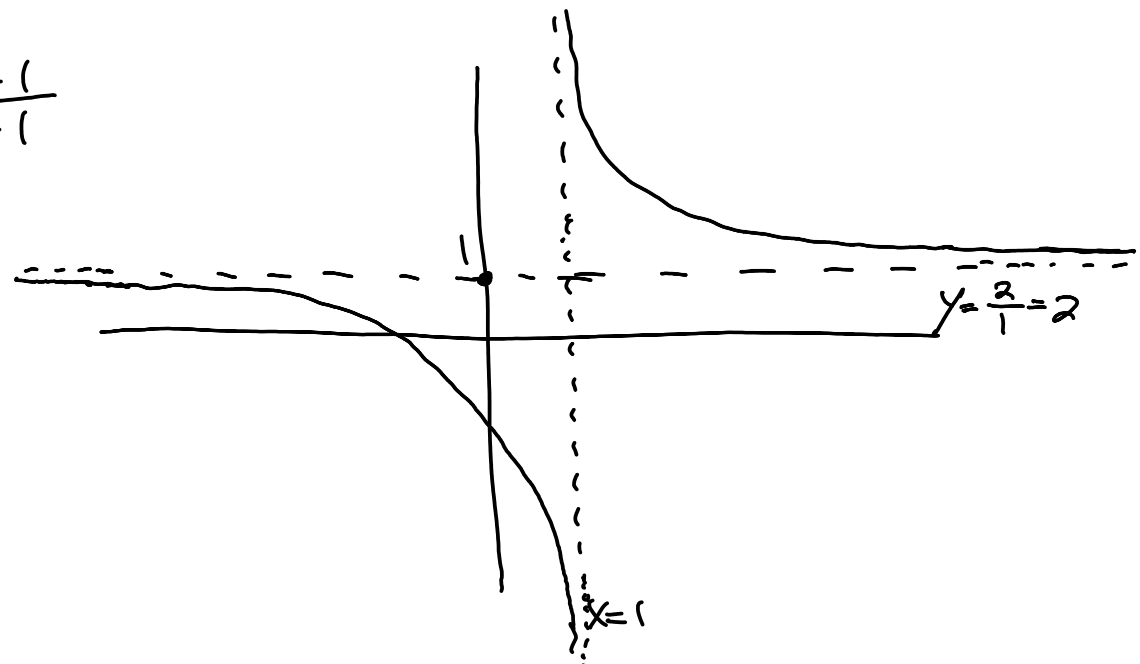


example $y = \frac{1}{x^2}$



(2) When degree(p(x)) = degree(q(x)) Then $y = \frac{ax^n + \cdots\cdots}{bx^n + \cdots\cdots\cdots}$ has The line $y = \frac{a}{b}$ is a "horizontal asymptote."

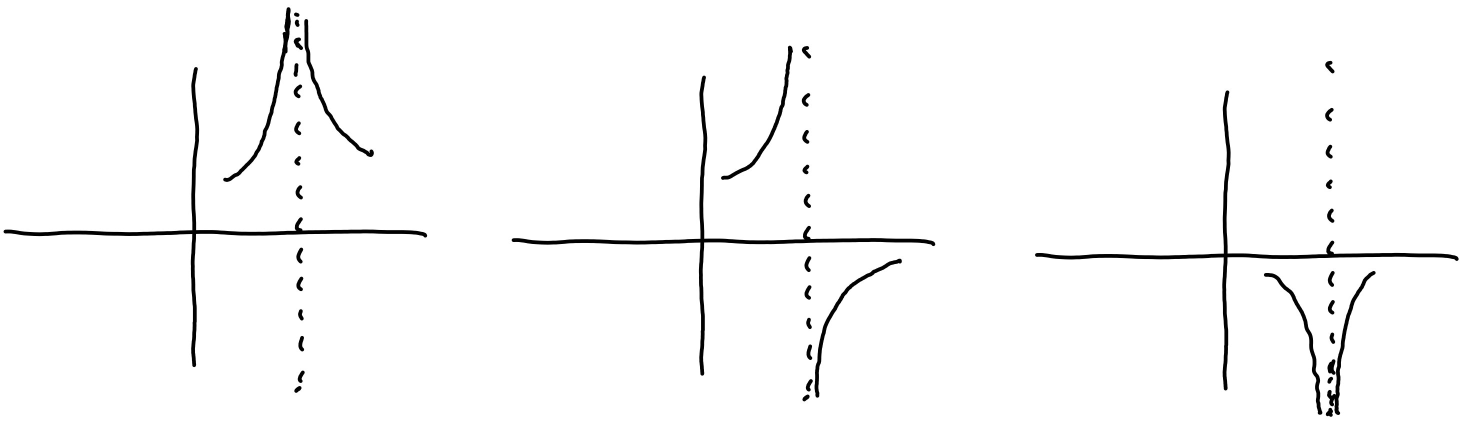
example $y = \frac{2x+1}{x-1}$



vertical asymptotes

given $y = \frac{p(x)}{q(x)}$, if r is a "zero" of The denominator $q(x)$, in other words, $q(r)=0$, Then as long as $p(r) \neq 0$ The vertical line $x=r$

is a vertical asymptote of $y=\frac{p(x)}{q(x)}$



<u>y-intercept</u>

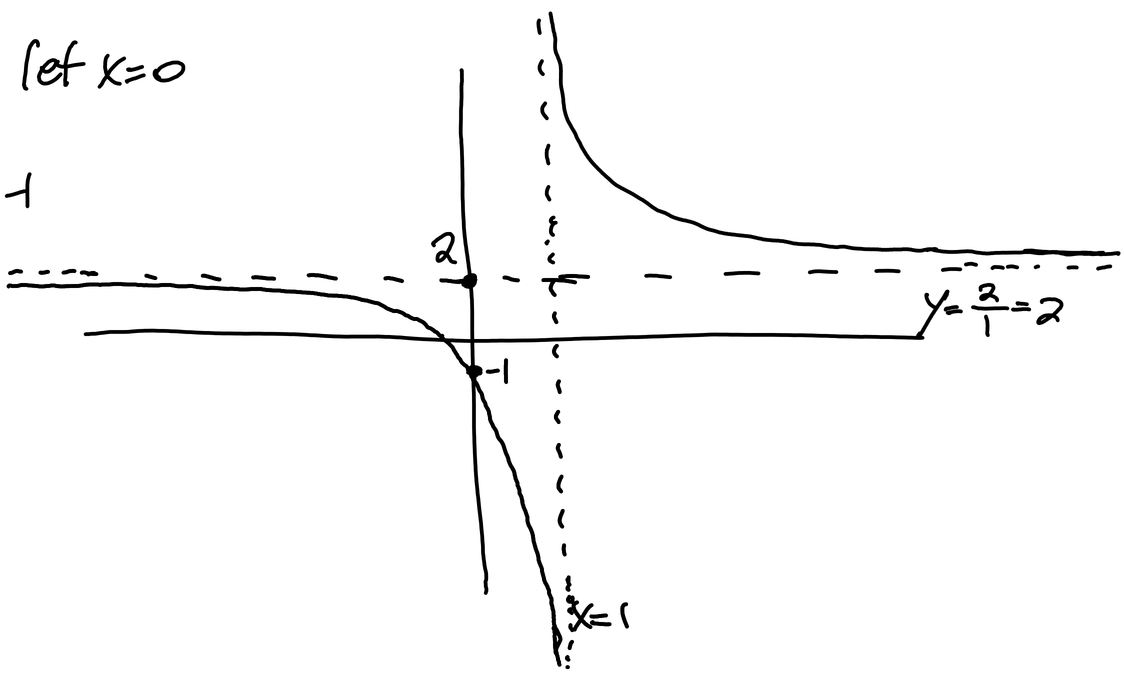
Occurs at $x=0$.

<u>example</u> $y=\frac{-x}{(x-1)^2}$ has y-intercept $y=\frac{-0}{(0-1)^2}\ \frac{0}{1}=0$

<u>example</u>

$y=\frac{2x+1}{x-1}$ let $x=0$

$y=\frac{0+1}{0-1}=\frac{1}{-1}=-1$



x-intercepts

for $y = \frac{p(x)}{q(x)}$ an x-intercept occurs at $x = a$

when $p(a) = 0$ and $q(a) \neq 0$.

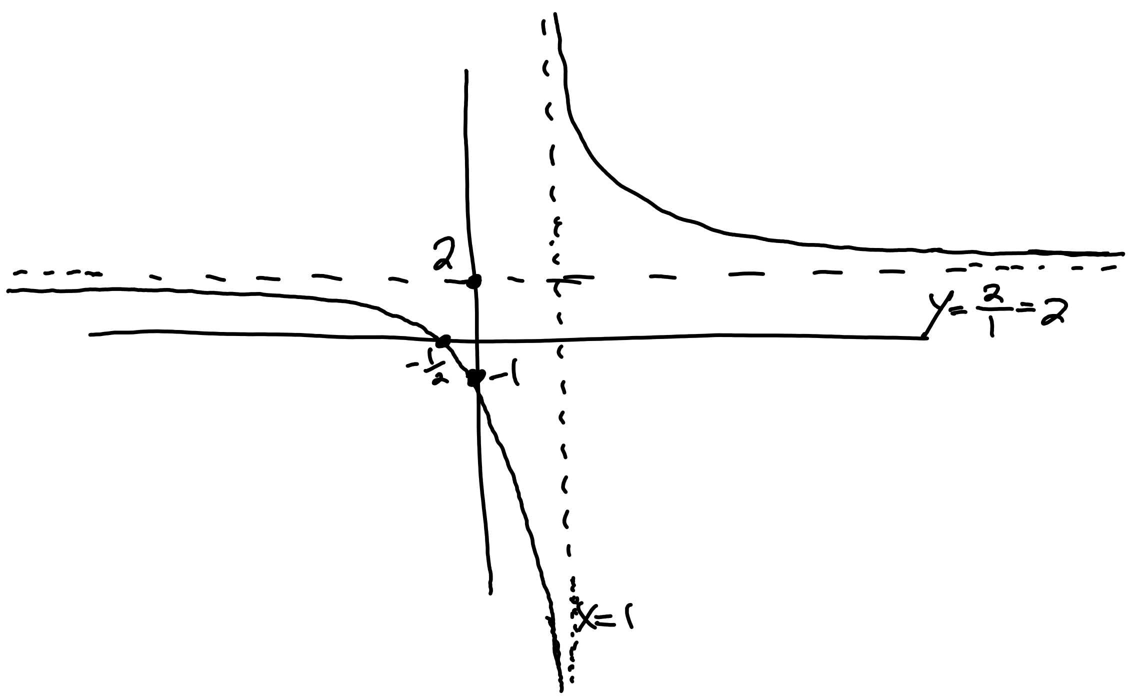
example

$$y = \frac{2x+1}{x-1}$$

numerator = 0

$2x + 1 = 0$

$x = \frac{-1}{2}$



positive/negative behavior

Take all of the x-values for x-intercepts and vertical asymptotes

and put them on a number line.

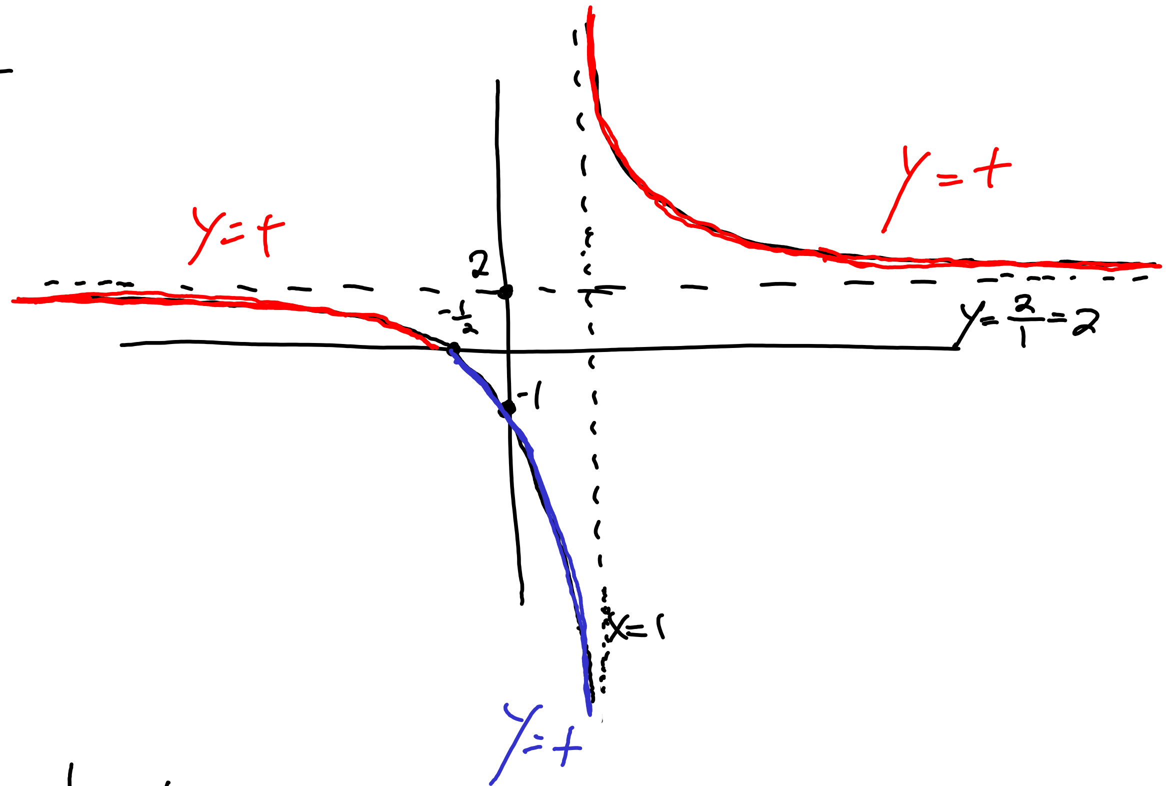
Then test intervals to see which have $f(x) = +$

and

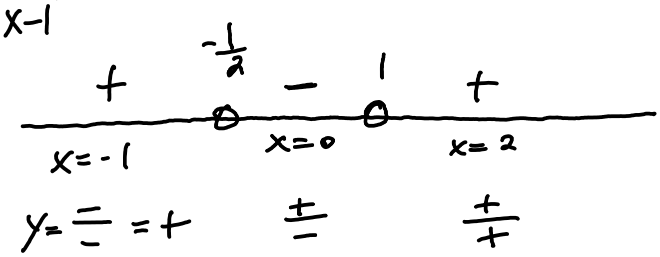
$f(x) = -$

example

$$y = \frac{2x+1}{x-1}$$



$$y = \frac{2x+1}{x-1}$$



Other assumptions

① The graph $y = \frac{p(x)}{q(x)}$ is smooth and unbroken except at vertical asymptotes.

③ $y = \frac{p(x)}{q(x)}$ wiggles as little as possible. In other words, no relative maximums unless they are forced.

<u>example</u> Sketch the graph of the Möbius function

$$y = \frac{4x-3}{x+1}$$

Label all asymptotes, x- and y-intercepts.

① <u>horizontal asymptote</u> $y = \frac{4}{1} = 4$

② <u>vertical asymptote</u> denominator $= 0$

$$x+1 = 0$$
$$x = -1$$

③ <u>y-intercept</u> set $x = 0$

$$y = \frac{0-3}{0+1} = -3$$

④ <u>x-intercept</u> numerator $= 0$

$$4x - 3 = 0$$
$$x = \frac{3}{4}$$

⑤ <u>positive/negative behavior</u>

$$y = \frac{4x-3}{x+1}$$

$+$		$-$		$+$
$x=-2$	-1	$x=0$	$\frac{3}{4}$	$x=1$
$\frac{-}{-}$		$\frac{-}{+}$		$\frac{+}{+}$

$$y = \frac{4x-3}{x+1}$$